A Precise Definition and Statistical Description of Firm Size

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Abstract

The paper shows how to estimate firm size more accurately other than by proxy. A precise definition of size is first derived from the two postulates required for the validity of financial ratios. Then, an estimation method for size is proposed. Evidence is also provided, supporting the feasibility of the proposed size estimate. Finally, potential applications are discussed.

Key-Words: Firm size, Ratio validity, Proportionate growth.

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Introduction

Firm size remains a poorly defined concept. Where the use of size is required by theory, empirical studies typically revert to using some proxy or other, such as the number of employees, Total Assets, Market Capitalisation or Sales. Firm size, in turn, is also viewed as a substitute for numerous theoretical constructs ranging from risk to liquidity or even political costs (Ball and Foster, 1982). As a result, firm size has been interpreted in many different ways, 'allowing it to explain everything, and thus nothing, at the same time' (Bujaki and Richardson, 1997).

The paper shows how to estimate size more accurately other than by proxy. Variables presently used as surrogates for size inevitably contain, besides this specific effect, some variability of their own. Such unwanted variability may distort or muddle measurement. Empirical research or other tasks where size is an explanatory variable will therefore benefit from the possibility of using a more accurate estimate. The analytical review of accounts using statistical methods or the automatic detection of atypical values in financial statements, are two amongst other application areas where an accurate size estimate will be beneficial as explained in the paper.

The proposed definition of size is derived from the two postulates underlying a widely used model, the financial ratio, where the effect of size is implicitly assumed. Ratios are viewed by managers, investors and analysts as capable of controlling for size so that comparisons may be made. The paper explores the premise that, if size can be controlled for by ratios, then it should be possible to estimate it. Specifically, the postulates underlying the successful removal of size by ratios should implicitly contain a definition of firm size. By identifying and using such a definition, it should be possible to estimate size. Evidence suggests that this reasoning leads to an accurate estimate of size.

Nowadays there is a widening consensus on the usefulness of financial ratios. Indeed, ratios have long been used by managers, investors and financial analysts and their usefulness is therefore validated by use. However, for more than thirty years, authors questioned the validity of ratios. Deakin (1976), for instance, noticed that the frequency distribution of ratios differs widely. As a result, he questioned the validity of analytical methods which assume the normal ratio data. Lev & Sunder (1979) suggested that the use of ratios was motivated by mere routine habits. These and other early authors paid little attention to the fact that variables from which ratios are constructed are often of a multiplicative type.[1] More recently, some authors began exploring the multiplicative case more in detail. McLeay (1986) has demonstrated that a fuller understanding of ratios can be achieved by taking account of the characteristics of the variables from which ratios are constructed. Trigueiros (1995) showed extensive evidence of the multiplicative character of accounting variables and offered a simple explanation for the variety of distributions found in ratios. Sudarsanam & Taffler (1995) found that non-proportionality in ratios is likely to be significant in the time series context but not in the cross section context. McLeay & Trigueiros (2002) showed that the multiplicative character of the variables from which financial ratios are constructed is a necessary condition of valid ratio usage.

The following sections demonstrate that the two postulates underlying the successful removal of size by ratios implicitly contain a definition of size. First, natural forms[2] are derived obeying such postulates. Then, based on such forms, size is medelled and tested. The paper also discusses potentially useful applications.

First Postulate: Proportionality

Ratios present one number as a representation of another, scaled up or down by a given factor. The Interest Cover ratio, for instance, is intended to show the number of times Earnings is greater than Interest; the Sales Margin ratio aims at expressing Earnings, the numerator, as a fraction of Sales, the denominator.

Financial analysis is just one of the many tasks where scaling is useful. Maps and other models, for instance, are also governed by a ratio showing the number of times any measurement in the representation is smaller than in reality. It is not surprising, therefore, that the first and most basic postulate of ratio analysis, the same as that governing maps, simply asserts that the scaling factor should remain constant. In maps, a constant scaling factor is applied to all measurements so that small objects are depicted as proportionally small representations and large objects are depicted as proportionally large representations. Similarly, in ratio analysis, a given ratio value is supposed to have the same financial meaning no matter the magnitudes being analysed. The first postulate of valid ratio usage is thus proportionality between the numerator of the ratio, X_1, and the denominator, X_2, so that X_1/X_2 is a constant scale (Whittington, 1980). In this case the rate of change of X_1 with respect to X_2 is also constant and similar to the ratio itself:

$$\frac{X_1}{X_2} = \frac{dX_1}{dX_2}$$

where dX_1, dX_2 are any related changes (across firms or in the same firm for different periods) in X_1 and X_2. Or alternatively,

$$\frac{dX_1}{X_1} = \frac{dX_2}{X_2}. \tag{1}$$

That is, the mechanism leading to proportional variables is scale invariance.

Scale invariance is indeed a necessary condition of valid ratio usage. For example, unless it is postulated that percent changes across firms of any sizes are the same for Current Assets and Current Liabilities, it becomes impossible to infer whether a Current Ratio above the standard is, for a large firm, attributable to the liquidity of that firm or simply to some characteristic of larger firms.

Scale invariance, in turn, implies a size mechanism. In order to see why, consider first that, for each ratio that can be validly used, equality (1) implies that there must exist a common source of variability influencing two of the items reported in a set of accounts, namely the numerator and the denominator of that ratio. Notice also that the validity of the ratio method as a whole rests on the validity of several widely used ratios. Since items used to form one ratio are often used to form other ratios as well, it follows that there must exist a common source of variability underlying most of the items found in published sets of accounts.[3]

Where items $X_1, X_2, \cdots X_N$ are proportional, there must exist a constant r such that

$$\frac{dX_1}{X_1} = \frac{dX_2}{X_2} = \cdots = \frac{dX_N}{X_N} = r. \tag{2}$$

Equality (2) is indeed required for the validity of the ratio method as a whole.

It is now easy to show that r, the common source of variability in $X_1, X_2, \cdots X_N$, possesses the attribute of a size effect. Consider two sets of accounts, a and b. In (2), differences between set b and set a are

$$\begin{aligned} dX_1 &= X_{1b} - X_{1a} \\ dX_2 &= X_{2b} - X_{2a} \\ &\vdots \\ dX_N &= X_{Nb} - X_{Na} \end{aligned}$$

and it is possible to express items in set b in terms of items in set a as

$$\begin{aligned} X_{1b} &= X_{1a}(1 + r_{ba}) \\ X_{2b} &= X_{2a}(1 + r_{ba}) \\ &\vdots \\ X_{Nb} &= X_{Na}(1 + r_{ba}) \end{aligned}$$

where r_{ba} is the relative difference whereby any item in b differs from the corresponding item in a. Structurally, that is, at the level of generative mechanisms, if one item is larger in b than in a, it follows that the other items in b will also be larger than the corresponding items in a. If Current Assets is larger in b than in a, then Sales must also be larger in b than in a and so on.

It is thus possible to say without ambiguity that set b is, as a whole, larger or smaller than set a. Now, wherever it is possible to rank two sets of accounts by size, it becomes also possible to rank any number of sets of accounts by size. The existence of a size mechanism underlying the postulate of proportionality is thus demonstrated.[4] If ratios are valid, therefore, it should be possible to estimate the size of a given set of accounts, relative to another set taken as the unit size.

Consider a set of accounts and, within it, a collection $X_1, X_2, \cdots X_N$ of items obeying proportionality. When an observer asserts that one firm is large or small, he or she is necessarily referring to some implicit pattern of size, which is taken as standard or unit for comparison. Similarly, in order to construct a usable size measurement it is first necessary to define the unit size set, $\mathcal{X}_1, \mathcal{X}_2, \cdots \mathcal{X}_N$ so that the rate r in (2) or, identically, a multiplier $S = 1 + r$, may be estimated against such unit. In the cross section context, for instance, it may make sense to let $\mathcal{X}_1, \mathcal{X}_2, \cdots \mathcal{X}_N$ be the set of industry median values of $X_1, X_2, \cdots X_N$ so that S measures size relative to the industry.

In general, the k^{th} item from the j^{th} set of accounts may thus be described as

$$X_{kj} = \mathcal{X}_k \, S_j \qquad \text{with} \qquad S_j = 1 + r_j. \tag{3}$$

S_j shows the number of times the firm publishing the j^{th} set of accounts is larger or smaller than the unit set. The level $\mathcal{X}_k$ is the value of item k in this unit set.

Notice that $\mathcal{X}_k$ and S_j are independent. In fact, $\mathcal{X}_k$ is a constant value relating solely to item k. S_j, in turn, is a characteristic of the set of accounts, affecting all of $X_1, X_2, \cdots X_N$ in set j. This separability of effects stems from proportionality and allows ratios to remove size. Indeed, any ratio formed with two items obeying (3) will cancel the size multiplier while showing the number of times by which their components differ in the unit set.[5]

Second Postulate: Size is a Proportionate Effect

The previous section highlighted the similarity between ratios and scales. It should be recognised, however, that ratios are, in other aspects, different from scales. First, in ratio analysis, the scaling factor is not a convention as in maps. Rather, it is supposed to reflect some pre-existing reality or 'standard': the natural relationship between two accounting variables (a norm), a goal to attain, a benchmark, or simply the ratio of a previous period.[6] Second, in ratio analysis any discrepancy that may be observed in relation to this pre-existing standard is considered as valuable information whereas in maps any such discrepancy would be viewed as an error.

It is a requirement of the valid use of ratios that these discrepancies are size independent. For a given ratio to be valid, the likelihood of a discrepancy of, say, 1% in relation to the standard, must be the same for small and large firms. Contrariwise the relevance of a 1% over performance or under performance would vary, depending on the size of the firm. This too would render ratios useless. Therefore, besides proportionality, another postulate of valid ratio usage is homeoscedasticity, that is, the distribution of the ratio must be independent of size.

As a corollary, relative changes in both the numerator and the denominator of the ratio are required to be size independent too.[7] Variables where relative changes are size independent are said to obey the Gibrat's Law. Indeed, the generation of items such as Sales or Assets is a process of accumulation obeying a multiplicative law of probabilities consistent with the Gibrat's Law.

This second postulate makes it possible to characterise the generation of accounting numbers under the effect of size. In the simplest case it may be assumed that such effect is a function of a variable, τ. From (2), the mechanism driving relative changes in the k^{th} item from the j^{th} set is

$$\frac{dX_{kj}}{X_{kj}} = r_j \, d\tau. \tag{4}$$

Specific sets of accounts, say, set j, are now characterised by what value τ may take. The first postulate (proportionality) is verified when $r_j d\tau$ is the same for items from the j^{th} set of accounts. The second postulate is verified when $r_j \, d\tau$ is independent of τ. For discrete τ a solution of (4) is $x_{kj} = \mathcal{X}_k[1 + r_j]^\tau$ whereas for continuous τ it is

$$x_{kj} = \mathcal{X}_k \exp[r_j \tau]. \tag{5}$$

In both cases x_{kj} is a realized X_{kj}; the level $\mathcal{X}_k$ is the value of x_{kj} for $\tau = 0$. In a time series context, r_j is the rate of growth of the j^{th} set of accounts in instant τ. In cross section, the meanings of r_j and τ are *a priori* less intuitive. Where r_j is taken as the constant rate of change denoting the variability of size in a given industry, then τ will measure, in a logarithmic scale, how far a set of accounts is from another set taken as the unit. In that case, $r_j \tau$ will have the meaning of a logarithmic size whereas $\exp[r_j \tau]$ is a size multiplier.

The above solutions are particular forms of (3), exhibiting the same separability of effects but where S_j, the size multiplier, is no longer undefined. Indeed, as a consequence of the second postulate, S_j in (3) has been shown to be an exponentiation of $r_j \tau$.

The Gibrat Law may lead to several types of random variables. Suppose, for instance, that x_{kj}, the observed k^{th} item in set j, is explained as

$$x_{kj} = \mathcal{X}_k \exp[r'_j \tau + z_{kj}], \tag{6}$$

where $r'_j\tau$ is the logarithmic size and z_{kj} accounts for unexplained variability. Now consider two realisations of (6) with similar $r'_j\tau$ (proportionality), where r'_j is independent of τ (homeoscedasticity) and the z_{kj} are independent and normally distributed with zero mean. Ratios of two of such items can be validly used. Indeed, variables obeying (6) illustrate the simplest case where the two postulates of ratio analysis are verified.

To finish, it is woth mentioning that equality (1) may not necessarily lead to (6). Indeed, an equally simple random variable that may also be derived from (1) is

$$x_{kj} = \mathcal{X}_k \exp[(r_j - \frac{\sigma^2}{2})\tau + z_{kj}], \tag{7}$$

for instance, in the context of stochastic time series (Tippett, 1991). Since the compounding effect is now influenced by σ, the standard deviation of z_{kj}, not just by size, variables obeying (7) are non-proportional in spite of being driven by mechanism (1). Thus (1) cannot express scale invariance in all possible cases. The mechanism capable of generating stochastic, scale invariant, items is obtained by equating two continuously compounding rates of change. Recall that r may be transformed into the corresponding continuously compounding rate r' by using $\log(r+1)$ instead of r. When this is applied to both sides of equality (1) it becomes

$$d\log X_2 = d\log X_1. \tag{8}$$

This equality is indeed capable of delivering stochastic proportionality. Therefore, rather than (4), a general mechanism capable of explaining the generation of proportionate numbers is

$$d\log X_{kj} = r'_j\, d\tau \tag{9}$$

where, due to the first postulate (proportionality), $r'_j\, d\tau$ is the same for items from the j^{th} set of accounts whereas, due to the second postulate (homeoscedasticity), r'_j is independent of τ. This is the continuously compounding version of the Gibrat's Law and, in the simplest (random) case, leads to (6).

How to Estimate Firm Size

Assume that the observed magnitude of item k from set j can be described as in (6) or, in an additive form, as

$$\log x_{kj} = \mu_k + \varsigma_j + z_{kj} \tag{10}$$

where $\mu_k = \log \mathcal{X}_k$ and $\varsigma_j = r_j' \tau$ is the logarithmic size.

Notice that (10) is basically an 'Analysis of Variance' formulation, i.e., a type of linear model aimed at explaining variability in terms of membership of discrete classes.[8] Indeed, in (10), $\log x_{kj}$ is explained by its membership of two classes, the item class, μ_k, and the set of accounts class, ς_j. The item class is a 'fixed' (deterministic) effect, as it denotes the fact that item k is one amongst the items making up the set of accounts and these are indeed fixed in number and in type. By contrast, the set of accounts class is a 'random' effect as it denotes the fact that set j is one of the (randomly selected) sets of accounts in the sample. Each of these two classes possesses several levels, namely, there can be as many levels of μ_k as items in the set of accounts. Similarly, there can be as many levels of ς_j as the sets of accounts in the sample.

In cross section, μ_k is the expected logarithmic item, estimated as the mean of the logarithms of that item across sets of accounts. Thus, in cross section $\mathcal{X}_k$ in (6) is the median item. The logarithmic size ς_j is the expected $\log x_{kj} - \mu_k$ for numbers in set j and its estimation is straightforward: given N numbers, all of them drawn from set j, (10) is first used to explain the variability of each of these numbers. The N formulations thus obtained are then added together. Since j (thus ς_j) is the same in all of these formulations, it is possible to write

$$\varsigma_j = \frac{1}{N} \sum_{k=1}^{N} (\log x_{kj} - \mu_k) - \frac{1}{N} \sum_{k=1}^{N} z_{kj}.$$

Any source of variability common to all $\log x_{kj}$ in j is, by construction, accounted for by ς_j. Therefore, even where correlation amongst some z_{kj} may exist, the term

$$\frac{1}{N} \sum_{k=1}^{N} z_{kj}$$

should tend to zero with an increasing N, leading to s_j, the estimate of

ς_j, as follows:

$$s_j = \frac{1}{N} \sum_{k=1}^{N} \left(\log x_{kj} - \mu_k \right). \tag{11}$$

In short, a well-behaved estimate of size may be obtained by averaging the logarithms of N appropriately adjusted items, drawn from the firm's actual set of accounts. Exact confidence intervals for s_j can also be obtained, the corresponding standard errors being t-distributed with $N-1$ degrees of freedom. Where μ_k is the expected logarithmic item (this is, in general, the case in cross section), ς_j has an expected value of zero.

The estimation of ς_j faces two obvious difficulties. First, some z_{kj} are correlated, which increases the standard error. Second, x_{kj} in (11) cannot be drawn from all possible numbers in a set of accounts because items such as Earnings, being the result of the subtraction of two other items, may take on negative values and cannot be transformed into logarithms. Such restriction may introduce a bias in the estimation of size and, by reducing the number of cases available, it further increases its standard error.

Empirical Testing of Hypotheses

The paper presents an exploratory data analysis supporting the hypotheses that size may be described as a random effect consistent with (10) and that size may be estimated using a subset of N items drawn from a set of accounts as in (11). The analysis is based on cross sections of sets of accounts available in the Worldscope disk.[9] Only reports issued by industrial firms based in the UK and denominated in Sterling were selected. Eight groups of industries were examined for five consecutive years (1993-1997) in a total of 40 different cross sections (Table 1). Although no other selection was made and no individual report was excluded from any group, one should bear in mind that, similarly to most of the commonly available databases of accounting reports, Worldscope only includes publicly quoted companies.

The reason for selecting the industrial groups in Table 1 stems from the interest in examining industries as different as possible while keeping the number of cases in each cross section as balanced as possible. The use of other industries also available would lead to some cross sections either being too large or too small in comparison to other cross sections.

	1993	1994	1995	1996	1997
Apparel and Textiles	59	57	56	64	62
Beverages and Food	55	59	59	68	60
Chemicals and Drugs	64	71	76	99	95
Electrical and Electronics	66	69	77	96	100
Machinery and Metal Manufacturers	86	91	88	101	90
Paper	33	36	38	44	37
Construction	57	57	58	60	62
Miscellaneous	81	83	87	104	101

Table 1: Industrial groups examined, number of firms per year.

CX	Cash and Short Term Investments
RCV	Receivables (net)
I	Total Inventories
FA	Property, Plant and Equipment (net)
E	Common Equity
NOE	Number of Employees
S	Net Sales or Revenues
CGS	Cost of Goods Sold (excl. Depreciation)
DPR	Depreciation, Depletion and Amortization of the year
INT	Interest Expense on Debt

Table 2: The 10 items selected for the estimation of size and abbreviations used.

Table 1 also shows the number of sets of accounts in each cross section.

Sets of accounts included in the Worldscope disk comprise 38 numerical items from the Balance Sheet and Profit and Loss Account. Items evidencing the number of employees and the cash flow from operations are also available. Table 2 shows the 10 items selected to estimate size. This selection stems solely from the fact that, for the considered industries, only these 10 items exhibit a tolerably small proportion of negative, zero or missing values.

For each of the 10 items selected, the frequency distributions of the 40 cross sections are first examined. The lognormal character of these distributions is evidenced by their high positive skewness and kurtosis, and by the regularity of the relationship between these two statistics (Aitchison and Brown, 1957). After a logarithmic transformation is applied, both skewness and kurtosis are subdue into normal values.

Item	CX	RCV	I	FA	E	NOE	S	CGS	DPR	INT
CX	0.97									
RCV	0.56	0.52								
I	0.54	0.53	0.67							
FA	0.55	0.48	0.51	0.61						
E	0.55	0.43	0.46	0.51	0.52					
NOE	0.47	0.39	0.42	0.43	0.36	0.45				
S	0.56	0.52	0.56	0.52	0.45	0.43	0.57			
CGS	0.55	0.53	0.58	0.52	0.45	0.43	0.58	0.61		
DPR	0.58	0.48	0.50	0.54	0.47	0.41	0.50	0.50	0.53	
INT	0.52	0.54	0.59	0.60	0.49	0.45	0.56	0.57	0.55	0.84

Table 3: Variance-covariance matrix, logarithms of items, paper industry, 1997.

Table 3 shows a typical variance-covariance matrix of logarithms of items. Co-variances are remarkably similar with no negative or zero co-variances. Matrices such as that in Table 3 denote a sizeable source of common variability. The logarithms of accounting numbers taken from the same set seem to be just replications of the same underlying effect (that is, size) to which item specific levels plus some randomness is added. This description adheres to (10) and can be observed in all cross sections.

The basic hypothesis of the paper is tested as follows: one variable is constructed, containing all the numbers from each of the cross sections, no matter the set of accounts or item they belong to. Then, a linear modelling algorithm[10] is used to specify a 'mixed model' formulation identical to (10), where the sums of squares attributed to effects μ_k and ς_j are tested against the residual sum of squares. If firm size is a sizeable statistical effect, then there should exist a significant source of variability associated with numbers from the same set of accounts regardless of the item they represent. An Analysis of Variance should thus reject the null hypothesis that these numbers add as much variability to the total as they would without the partition into sets of accounts.

Results clearly support the hypothesis. In all of the cross sections, within set mean squares are much smaller than corresponding between set mean squares. The size effect, ς_j, is highly significant ($P < 0.001$) in all analyses. The μ_k effects are also highly significant ($P < 0.001$). Jointly, size and item effects explain between 82% and 92% of the variability of accounting numbers. This is a clear indication of the adequacy

Cross section	Effect	1991	1992	1993	1994	1995
Apparel and Textiles	μ_k	50.0%	49.0%	49.1%	48.2%	50.9%
	ς_j	38.8%	37.9%	37.8%	37.1%	35.0%
Beverages and Food	μ_k	34.2%	36.4%	37.1%	36.8%	37.9%
	ς_j	55.4%	52.4%	50.6%	51.4%	50.0%
Chemicals and Drugs	μ_k	29.3%	29.7%	31.3%	29.9%	31.8%
	ς_j	58.7%	56.7%	57.5%	51.6%	49.7%
Electrical and	μ_k	39.3%	41.5%	41.5%	32.3%	37.6%
Electronics	ς_j	48.2%	47.8%	47.6%	54.6%	47.2%
Automotive, Machinery	μ_k	42.3%	43.9%	44.7%	43.2%	45.3%
and Metal Manufacturers	ς_j	47.2%	47.1%	47.1%	45.5%	40.4%
Paper and Publishing	μ_k	42.9%	46.5%	48.8%	47.4%	46.8%
	ς_j	46.2%	42.6%	41.3%	38.3%	39.7%
Construction	μ_k	34.5%	35.7%	34.5%	35.7%	39.1%
	ς_j	54.6%	54.1%	54.6%	52.8%	49.2%
Miscellaneous	μ_k	39.3%	42.3%	41.5%	39.1%	39.4%
	ς_j	45.9%	43.9%	44.5%	44.7%	43.1%

Table 4: Proportion of variability explained by each effect in the Analysis of Variance. μ_k is the item effect, ς_j is the size effect.

of (10). Table 4 shows the proportion of variability explained by each of these effects.[11]

The frequency distributions of residuals are symmetrical but leptokurtic in all cross sections. Significant correlation is observed between the residuals of Sales, Inventory and Receivables but not between the residuals and size. The distributions of the ς_j, being the averages of 10 random numbers, strictly adhere to normality.

The paper now examines in detail how the characteristics of the size estimate are affected by the number of items employed. According to (11), size is estimated as an average of mean-adjusted logarithms of items. In the present case, the items available for averaging are those in Table 2. Accordingly, a collection $s_1, s_2, \cdots, s_{10}$ of size estimates is built, each incorporating an increasing number of items. Schematically,

$$\begin{aligned} s_1 &= CX, \\ s_2 &= 1/2\,(CX + RCV), \\ s_3 &= 1/3\,(CX + RCV + I), \text{ and so on until } s_{10}. \end{aligned}$$

The collection $s_1, s_2, \cdots, s_{10}$ of size estimates is used, in the first place, to observe how the entrance of each new item affects the variance of the estimate. If the conditions required for successful use of (11) are verified, namely, if size is clearly dominant, then the cross section variability of the estimate should tend to decrease, eventually stabilising when all unexplained variability has been 'diversified'. Figure 1 compares the variances of $s_1, s_2, \cdots, s_{10}$ for the cross sections considered. The tendency towards decreasing variance is apparent in all cases. A trend towards stabilisation is also apparent.

On the other hand, most of this decrease in variance takes place when introducing the second item. As such, it may be speculated that the inclusion of more than two items would add little in terms of diversification. The Analysis of Variance algorithm is thus used again to determine whether several items are needed to obtain a good size estimate or, by contrast, two or three items may suffice. The experiment consists of examining how the significance of ς_j evolves when s_1, s_2 and so on, are, in succession, subtracted from $\log x_{kj}$ before being introduced into the model. If diversification is at work, the progressive build up of the size effect in $s_1, s_2, \cdots, s_{10}$ should determine an also progressive loss of significance of the size effect in $\log x_{kj} - s_1, \cdots, \log x_{kj} - s_{10}$.

Results evidence the gradual nature of diversification. When s_1, s_2 or s_3 are subtracted from $\log x_{kj}$, the ς_j remain significant for all cross sections. When s_4, s_5 or s_6 are subtracted from $\log x_{kj}$, this effect progressively loses significance, yielding non-significant for all cross sections ($P > 0.05$) only when s_6 is used. When all 10 items are included in the estimate, $P > 0.99$ for all cross sections. The progressive build up of size is apparent.

The standard errors and the two 95% confidence intervals are also examined. For logarithmic estimates ranging from -2 to +2, (size multipliers ranging from 0.01 times to 100 times) errors are stable with average values of 0.25 (1.78 times). There is no correlation between errors and size. These errors can be further reduced in samples drawn from homogeneous industries where more than 10 items may be used for estimating size.

Potential Applications

It was mentioned that empirical research where size is an explanatory variable will benefit from the increase in specificity brought about by an

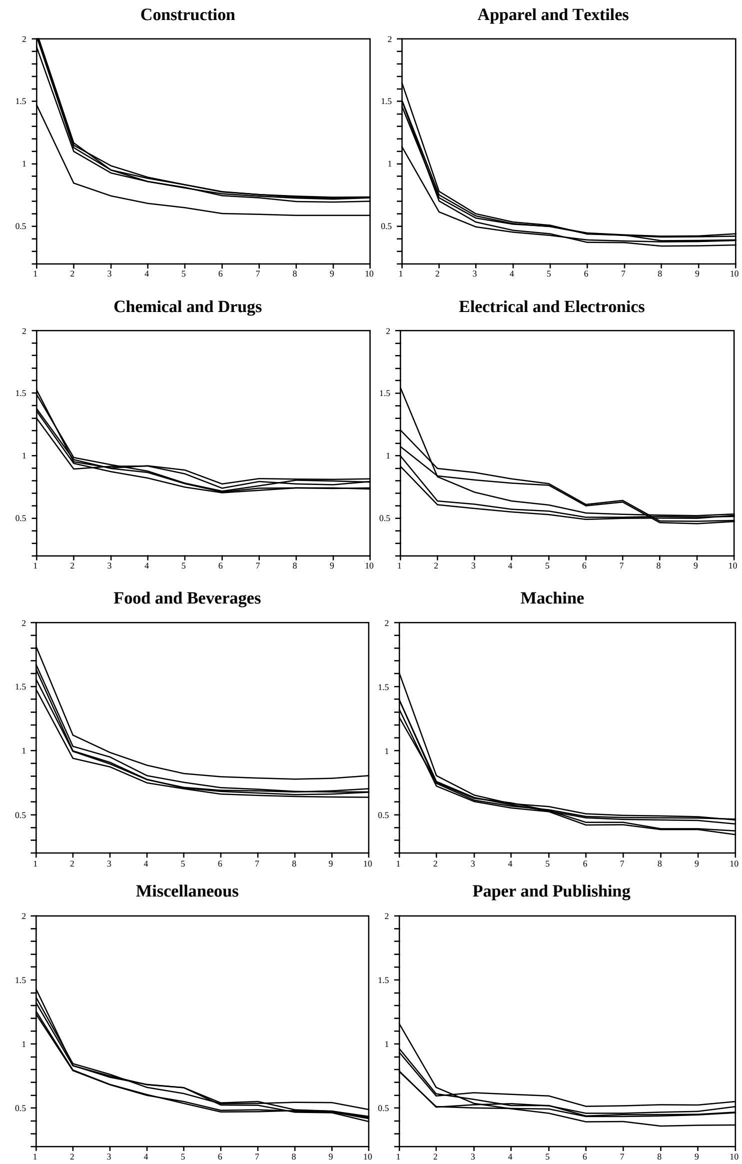

Figure 1: The decrease in variance of the size estimate when items are included, one by one, in the estimation. The X-axis indicates the number of items included and the Y-axis indicates variance of s. Each line depicts one of the five years examined.

accurate size estimate. Indeed, the possibility of building an estimate able to closely approach size may lead to a better understanding of some important constructs.

Besides such direct use of s_j, other promising applications relate to the residual z_{kj} in (10). These residuals are, in a logarithmic scale, industry adjusted ratios formed with a given item in the numerator and size in the denominator. Using z_{kj}, it is therefore possible to compare the characteristics of firms of different sizes in much the same way ratios do, but including in the analysis one item at a time.

Ratios, while removing size, always compare two items. Indeed, most ratios explicitly seek this comparison of two magnitudes. For example, the Return on Capital or the Interest Cover ratios are useful because of the comparison that is made between two items, not because size is removed. There are, however, situations where an analysis based on size adjusted items may be useful. For instance, it may be interesting to rank firms in a given industry by Fixed Costs or by Long Term Debt.

Fraud detection tools, as well as the analytical reviewing of accounts using statistical methods, being based on the detection of abnormally large deviations from expectation, will also benefit from the use of an accurate size estimate. Surely, such an estimate must be the first step when performing these tasks; and any residual z_{kj} in (10) exhibiting a magnitude above or below three standard deviations, may be seen as an abnormal case.

Concluding Remarks

The paper has introduced a firm size estimate defined as the random effect underpinning items from the same set of accounts. Empirical evidence suggests that this effect is indeed significant, explaining a large proportion of the variability of accounting numbers.

The paper uses a simplified approach, consistent with the premise that the financial ratio, itself a simplified model, is valid. Surprisingly, such approach seems to be accurate enough to deal with some important situations. Namely, the proposed methodology is likely to be readily applicable to cross sections of accounting reports drawn from commercially available databases. Where 10 or more items are suitable for the estimation of size, the standard error is affordable in most application areas.

The developments just presented also provide a more accurate description of accounting variables. Items that form the Balance Sheet and the Profit and Loss Account have different origins and different characteristics but, to the extent that they are issued by the same firm during the same period, they are indeed generated under the same size influence. Moreover, such influence is paramount as shown in the paper. When modelling accounting numbers this fact should no longer be ignored.

Some difficulties in implementing the methodology are expected, not in cross section but in the time series context where the size estimate, s, has the meaning of a rate of growth. Fixed costs, for instance, may introduce non-proportionality amongst items. The non-stationary character of these series is another obstacle, especially for more than two periods. Therefore, although the paper stresses the parallelism between the estimation of size and that of growth, it is clear that it will not be easy to extend such parallelism to practice.

Notes

1. Multiplicative variables are generated by a multiplicative rather than by an additive law of probabilities. A typically multiplicative variable is the lognormal. Ijiri and Simon (1977) mention mechanisms and assumptions leading to the most common types of multiplicative variables.

2. Natural forms describe mechanisms or structures rather than observations. Uppercase letters and the term *mechanism* are used throughout to refer to natural forms.

3. Sales, for instance, is used by analysts to form Margin, Turnover and Profitability ratios together with items such as Total Capital Employed or Total Assets, Cost of Goods Sold and Gross Profits. Total Capital Employed is in turn used to form Leverage and other ratios. Cost of Goods Sold is used to form the popular Inventory Turnover ratio and other ratios. Consider, for instance, Sales (S), Costs of Goods Sold (C) and Total Capital Employed (T). Indeed, where

$$\frac{dS}{S} = \frac{dC}{C} \text{ and } \frac{dC}{C} = \frac{dT}{T} \text{ then } \frac{dS}{S} = \frac{dT}{T}.$$

   Williamson (1984) identified 11 ratios used by *Fortune 500* companies as part of their annual reports. Sales appears in 4 of these ratios. Indeed, amongst this set of 11 ratios, only 2 of them did not share components with other ratios.

4. It may be argued that, while this reasoning contemplates one source of variability only, the numerical information made available by firms probably is multi-dimensional, possessing several sources of variability. Actually, it is irrelevant whether such sources of variability are one or many. The issue here is not whether there exists one or several dimensions but whether the largest dimension is paramount. Evidence presented by the paper strongly suggests that size is indeed a clearly dominant dimension and this explains why observers experience no dificulty in identifying firms as large or small.

5. Thence the close relationship between ratio standards and the unit size set: individual ratio standards pre-determine the unit size set down to one degree of freedom.

6. What makes ratios potentially invalid is this comparison that is made with the pre-existing standard. Any financial indicator whose full interpretation would not require a comparison, thus being a fundamental magnitude in its own right, would be neither valid nor invalid. Only a measurement can be invalid. The problem of the validity of ratios is thus circumscribed to cases involving a comparison with a standard.

7. Changes $d\,(y/x)$ experienced by the ratio y/x are (Tippett, 1990)

$$d\,\frac{y}{x} = \frac{y}{x}\left(\frac{dy}{y} - \frac{dx}{x}\right).$$

   This, together with the postulate that valid ratios are size independent, leads to the conclusion that $d\,(y/x)$ is size independent with generality only where both dy/y and dx/x are size independent.

8. When compared to a typical Analysis of Variance, (10) is a limited case. Sets of accounts contain only one item of each type. Therefore, no interaction exists between μ_k and ς_j and these effects can be estimated separately. In (10), the ground mean or intercept, M, is implicit in the item effect, i.e., $\mu_k = M + \delta_k$. Its explicit inclusion would be confusing in the present context (see, e.g., Snedecor and Cochran, 1965).

9. The Worldscope disk is part of the Global Access research tool by Thomson.

10. GLM from SPSS. The design includes only the two main effects, μ_k and ς_j where ς_j is explicitly declared a random effect. As mentioned, the intercept term is excluded from the design. 'Type III' method for partitioning sums of squares is used.

11. The degrees of freedom associated with the fixed effect are 9, occasionally 8. The degrees of freedom associated with the random effect range from 103 to 32. Proportions in Table 4 were not corrected for degrees of freedom.

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