Forecasting Methods / Métodos de Previsão

Week 8 - Time Series Models

ISCTE - IUL, Gestão, Econ, Fin, Contab.

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Forecasting Methods - Time Series Models

- Time Series Analysis techniques
  - involves consideration of historical data, and obtaining future estimates based on past values
  - assume that what has occurred in the past will continue to occur in the future
  - relate the forecast to only one factor–time
  - includes: moving average and exponential smoothing models
  - Moving Averages and Exponential Smoothing are short range techniques. They produce forecast for the next period
  - Trend equations are used for much longer time horizons.
  - More than one forecasting techniques might be used to increase confidence

Forecasting Methods - Time Series Models

- A **time series**: time-ordered sequence of observations taken at regular intervals over a period of time

$$Y_1, Y_2, ..., Y_t$$

- **Examples**: daily stock price, monthly sales, annual revenue, etc.
- **Components** (Types of Variations) of a Time Series
  - *Trend* (T) – long term upward or downward movement
  - *Seasonality* (S) – the pattern that occurs every year (short-term regular variations in data)
  - *Cycles* (C) – the pattern that occurs over a period of years (wavelike variations of long-term not caused by seasonal variation; effect of the economy)
  - *Random variations* (ε)– caused by chance and unusual events

Forecasting Methods - Time Series Models

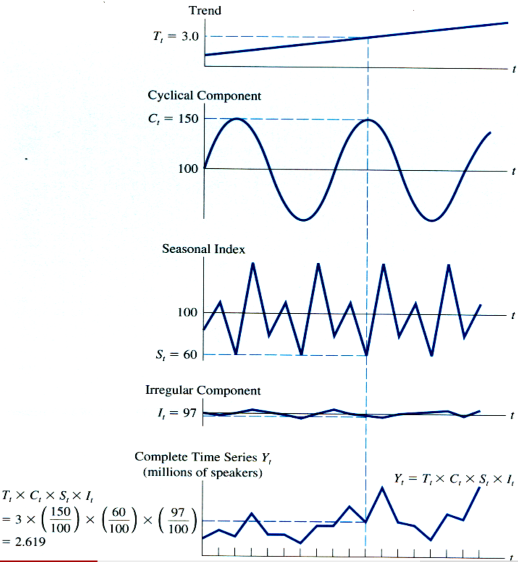
- A time series can be broken down into its individual components. Two approaches:
  - **Multiplicative decomposition**

    $$\boxed{\textbf{Forecast} = \textbf{Trend x Seasonality x Cycles x Random}}$$

  - **Additive decomposition**

    $$\boxed{\textbf{Forecast} = \textbf{Trend} + \textbf{Seasonality} + \textbf{Cycles} + \textbf{Random}}$$

Forecasting Methods - Time Series Models



Forecasting Methods - Time Series Models

- Stationary and Nonstationary Time Series Data
  - If a time series has an upward or downward trend, it is **nonstationary**
  - If it has no trend, it is **stationary**
- **Stationary Model Assumptions**
  - Assumes item forecasted will stay steady over time (constant mean; random variation only)
  - Techniques will smooth out short-term irregularities
  - Forecast for period $(t+1)$ is equal to forecast for period $(t+k)$; the forecast is revised only when new data becomes available.
- Stationary Model Types
  - Naïve Forecast
  - Moving Average
  - Weighted Moving Average
  - Exponential Smoothing

Forecasting Methods - Time Series Models

- A forecast is rarely completely accurate.
- Forecasts will usually deviate from the actual demand.
- The difference between the forecast and the actual is the forecast error.
- The objective of forecasting is to make the forecasting error as slight as possible.
- A large degree of error may indicate that either the forecasting technique is the wrong one or it needs to be adjusted by changing its parameters.

Forecasting Methods - Time Series Models

- Forecasting Performance (error): measures how accurate the forecast was
- For time period t:
  Forecast error = Actual value – Forecast value

$$F_e = A_t - F_t$$

- **Mean Forecast Error** (MFE or **Bias**): the arithmetic sum of the errors (average deviation of forecast from actual)
- **Mean Absolute Deviation** (MAD), Measures average absolute deviation of forecast from actual (T - the number of time periods)

$$MAD = \frac{\sum_{t=1}^{T} |A_t - F_t|}{T}$$

- **Mean Square Error** (MSE): measures variance of forecast error

$$MSE = \frac{\sum_{t=1}^{T} (A_t - F_t)^2}{T}$$

- **Mean Absolute Percentage Error** (MAPE): measures absolute error as a percentage of the forecast

$$MAPE = 100 \frac{\sum_{t=1}^{T} \left| \frac{A_t - F_t}{A_T} \right|}{T}$$

Forecasting Methods - Time Series Models

- **Bias, MAD, and MAPE** - typically used for time series
- Compare the accuracy of alternative forecasting methods using MAD and MSE (determine which method yields the lowest MAD or MSE for a given set of data).
- Want MFE to be as close to zero as possible – minimum bias
- A large positive (negative) MFE means that the forecast is undershooting (overshooting) the actual observations
- Note that zero MFE does not imply that forecasts are perfect (no error) – only that mean is "on target"

Forecasting Methods - Time Series Models

- The **Naïve Model**: the simplest forecasting technique
- Uses a single previous value of a time series as the basis of a forecast.
- A naive forecast for any period simply projects the previous period's actual value
- If it were true, future will always be the same as the past: whatever happened last period will happen again this time
- Virtually no cost; Data analysis is nonexistent;
- Easily understandable;
- Cannot provide high accuracy
- Provides a baseline to measure other models

- **Uses for Naive Forecasts:**
  - Stable time series data: Forecast is the same as the last actual observation

$$F_t = A_{t-1}$$

  - Seasonal variations: Forecast is the same as the last actual observation when we were in the same point in the cycle, where a cycle lasts n periods.

$$\begin{aligned} F_t &= A_{t-n} \\ F_t &= A_{t-4}: \text{ quarterly data; or } \quad F_t = A_{t-12}: \text{ monthly data} \end{aligned}$$

  - Data with trends: There is constant trend, the change from $(t-2)$ to $(t-1)$ will be exactly as the change from $(t-1)$ to (t)

$$F_t = A_{t-1} + (A_{t-1} - A_{t-2})$$

Forecasting Methods - Time Series Models

- Example

Wallace Garden Supply

Forecasting

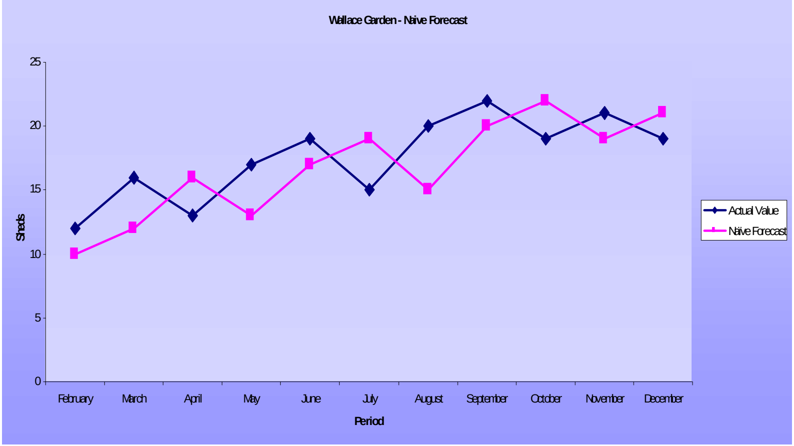
Storage Shed Sales

Period	Actual Value	Naïve Forecast	Error	Absolute Error	Percent Error	Squared Error
January	10	N/A				
February	12	10	2	2	16.67%	4.0
March	16	12	4	4	25.00%	16.0
April	13	16	-3	3	23.08%	9.0
May	17	13	4	4	23.53%	16.0
June	19	17	2	2	10.53%	4.0
July	15	19	-4	4	26.67%	16.0
August	20	15	5	5	25.00%	25.0
September	22	20	2	2	9.09%	4.0
October	19	22	-3	3	15.79%	9.0
November	21	19	2	2	9.52%	4.0
December	19	21	-2	2	10.53%	4.0
			0.818	3	17.76%	10.091
			BIAS	**MAD**	**MAPE**	**MSE**

Standard Error (Square Root of MSE) = 3.176619

Forecasting Methods - Time Series Models

- Example



Forecasting Methods - Time Series Models

- **Moving Average Method** (MA): Naïve methods just trace the actual data with a lag of one period, $F_t = A_{t-1}$, they don't smooth
- Averaging (over time) techniques are used to smooth variations in the data.
- Average most current values to predict future outcomes. The trend-cycle can be estimated by smoothing the series to reduce random variation.
- The forecast is the average of the last n observations of the time series.

$$F_{t+1} = \frac{Y_t + Y_{t-1} + ... + Y_{t-n+1}}{n}$$

- Note that the n past observations are equally weighted.
- Issues with moving average forecasts:
  - All n past observations treated equally;
  - Observations older than n are not included at all;
  - Requires that n past observations be retained;

Forecasting Methods - Time Series Models

- Example

Wallace Garden Supply
Forecasting

Storage Shed Sales

Period	Actual Value	Three-Month Moving Averages								
January	10									
February	12									
March	16									
April	13	10	+	12	+	16	/	3	=	12.67
May	17	12	+	16	+	13	/	3	=	13.67
June	19	16	+	13	+	17	/	3	=	15.33
July	15	13	+	17	+	19	/	3	=	16.33
August	20	17	+	19	+	15	/	3	=	17.00
September	22	19	+	15	+	20	/	3	=	18.00
October	19	15	+	20	+	22	/	3	=	19.00
November	21	20	+	22	+	19	/	3	=	20.33
December	19	22	+	19	+	21	/	3	=	20.67

- Example

Wallace Garden Supply

Forecasting 3 period moving average

Actual Value - Forecast

Input Data

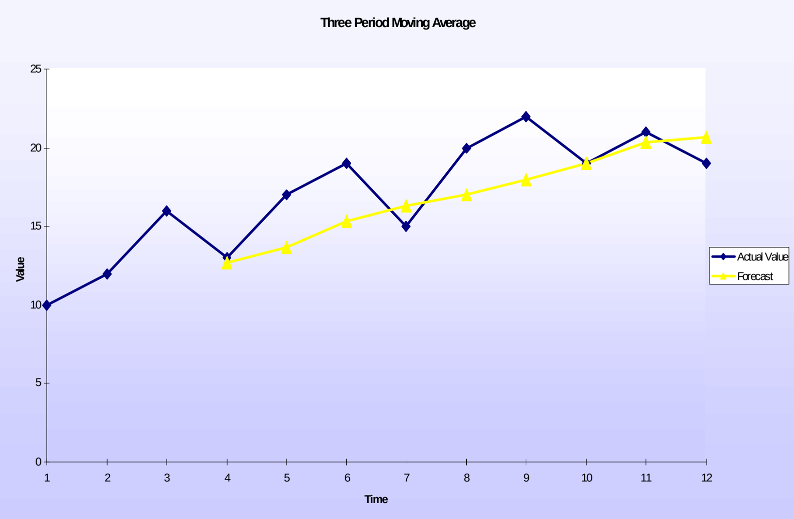
Period	Actual Value
Month 1	10
Month 2	12
Month 3	16
Month 4	13
Month 5	17
Month 6	19
Month 7	15
Month 8	20
Month 9	22
Month 10	19
Month 11	21
Month 12	19
Next period	**19.667**

Forecast Error Analysis

	Forecast	Error	Absolute error	Squared error	Absolute % error
	12.667	0.333	0.333	0.111	2.56%
	13.667	3.333	3.333	11.111	19.61%
	15.333	3.667	3.667	13.444	19.30%
	16.333	-1.333	1.333	1.778	8.89%
	17.000	3.000	3.000	9.000	15.00%
	18.000	4.000	4.000	16.000	18.18%
	19.000	0.000	0.000	0.000	0.00%
	20.333	0.667	0.667	0.444	3.17%
	20.667	-1.667	1.667	2.778	8.77%
Average		**1.333**	**2.000**	**6.074**	**10.61%**
		BIAS	**MAD**	**MSE**	**MAPE**

Forecasting Methods - Time Series Models

- Example



Forecasting Methods - Time Series Models

- MA Advantage = Easy to compute and easy to understand
- MA Disadvantage = All values in the average are weighted equally
- This technique derives its name from the fact that as each new actual values becomes available, the forecast is updated by adding the newest value and dropping the oldest and the recalculating the average.

Forecasting Methods - Time Series Models

- **Weighted Moving Average** (WMA): In the moving averages method, each observation in the MA calculation receives the *same weight*. One variation, known as weighted moving averages, involves selecting a *different weight* for each data value and then computing a weighted average of the most recent m values as the forecast.
- In most cases, the most recent observation receives the most weight, and the weight decreases for older data values (most recent observations must be better indicators of the future than older observations).
- Note that for the WMA the sum of each weights is equal to 1.
- The larger the n the more stable the forecast.
- A 2-period model will be more responsive to change.
- We don't want to chase outliers. But we don't want to take forever to correct for a real change. We must balance stability with responsiveness

Forecasting Methods - Time Series Models

- The Weighted Moving Average Method: historical values of the time series are assigned different weights when performing the forecast

$$
\begin{aligned}
F_{t+1} &= \text{Weighted sum of last n demands} \\
&= w_1 A_t + w_2 A_{t-1} + ... + w_n A_{t-n+1}
\end{aligned}
$$

where

F_{t+1} = forecast for Period $t+1$
n = number of periods used in determining the moving average
w = weights assigned to Period i (with $\sum w_i = 1$)
A_t = actual demand in Period t

Forecasting Methods - Time Series Models

- Example

Wallace Garden Supply

Forecasting

Storage Shed Sales

Period	Actual Value	Weights	Three-Month Weighted Moving Averages								
January	10	0.222									
February	12	0.593									
March	16	0.185									
April	13		2.2	+	7.1	+	3	/	1	=	12.298
May	17		2.7	+	9.5	+	2.4	/	1	=	14.556
June	19		3.5	+	7.7	+	3.2	/	1	=	14.407
July	15		2.9	+	10	+	3.5	/	1	=	16.484
August	20		3.8	+	11	+	2.8	/	1	=	17.814
September	22		4.2	+	8.9	+	3.7	/	1	=	16.815
October	19		3.3	+	12	+	4.1	/	1	=	19.262
November	21		4.4	+	13	+	3.5	/	1	=	21.000
December	19		4.9	+	11	+	3.9	/	1	=	20.036

Next period **20.185**

Sum of weights = **1.000**

Forecasting Methods - Time Series Models

- Example

Wallace Garden Supply
Forecasting 3 period weighted moving average

Input Data

Period	Actual value	Weights
Month 1	10	0.222
Month 2	12	0.593
Month 3	16	0.185
Month 4	13	
Month 5	17	
Month 6	19	
Month 7	15	
Month 8	20	
Month 9	22	
Month 10	19	
Month 11	21	
Month 12	19	

Next period **20.185**

Sum of weights = **1.000**

Forecast Error Analysis

Forecast	Error	Absolute error	Squared error	Absolute % error
12.298	0.702	0.702	0.492	5.40%
14.556	2.444	2.444	5.971	14.37%
14.407	4.593	4.593	21.093	24.17%
16.484	-1.484	1.484	2.202	9.89%
17.814	2.186	2.186	4.776	10.93%
16.815	5.185	5.185	26.889	23.57%
19.262	-0.262	0.262	0.069	1.38%
21.000	0.000	0.000	0.000	0.00%
20.036	-1.036	1.036	1.074	5.45%
Average	**1.988**	**6.952**	**6.952**	**10.57%**
	BIAS	**MAD**	**MSE**	**MAPE**

Forecasting Methods - Time Series Models

- Exponential smoothing is one of the more popular and frequently used forecasting techniques, the reasons being:
  - It requires minimal data.
  - The mathematics of the technique are easy to understand by management.
  - Most importantly, exponential smoothing has a good track record of success.
- Simple Exponential Smoothing works well with data that is "moving sideways" (stationary)
- Must be adapted for data series which exhibit a definite trend
- Must be further adapted for data series which exhibit seasonal patterns
- This technique is used for short term forecasting.

Forecasting Methods - Time Series Models

- **Exponential Smoothing: Single Exponential Smoothing** (SES o EWMA)
- SES is a special case of the WMA method in which we select only one weight, α, the weight for the most recent observation.
- Estimate next outcome with a weighted combination of the forecast for previous period and the most recent outcome
- Assumptions: No trend and exponentially declining weight given to past observations
- The weights for the other data values are computed automatically and become exponentially smaller as the observations move farther into the past.

$$\begin{aligned} F_{t+1} &= \alpha Y_t + (1-\alpha) F_t = F_t + \alpha (Y_t - F_t) \\ Y_t &= \text{time series value, } F_t = \text{ fitted (forecasted) value} \\ \alpha &= \text{ weight, } 0 \leq \alpha \leq 1 \end{aligned}$$

- or:
  $\boxed{\textbf{Forecast today} = \textbf{Forecast yesterday} + \alpha(\textbf{Forecast error yesterday})}$
- Each new forecast is equal to the previous forecast plus a percentage of the previous error.
- The initial value for the smoothing recursive process can affect the quality of the forecasts for many observations. In practice, when there are many leading observations prior to a crucial actual forecast, the initial value will not affect that forecast by much, since its effect will have long "faded" from the smoothed series.

Forecasting Methods - Time Series Models

- When applied recursively to each successive observation in the series, each new smoothed value (forecast) is computed as the weighted average of the current observation and the previous smoothed observation.
- Each smoothed value is the weighted average of the previous observations, where the weights decrease exponentially depending on the value of parameter α.
    - If $\alpha = 1$: Previous observations are ignored entirely (short memory).
    - If $\alpha = 0$: Current observation is ignored entirely (long memory).
- The most straightforward way of evaluating the accuracy of the forecasts based on a particular α value is to simply plot the observed values and the one-step-ahead forecasts.

Forecasting Methods - Time Series Models

- More complex ES models (double ES and Winters' method), have been developed to accommodate time series with trend and seasonal components.
- The general idea here is that forecasts are not only computed from consecutive previous observations (as in SES), but an independent (smoothed) trend and seasonal component can be added.
- The quickness of forecast adjustment to error is determined by the smoothing constant.
- The closer the alpha is to zero, the slower the forecast will be to adjust to forecast errors.
- Conversely, the closer the value of alpha is to 1.00, the greater the responsiveness to the actual observations and the less the smoothing
- Select a smoothing constant that balances the benefits of responding to real changes if and when they occur.

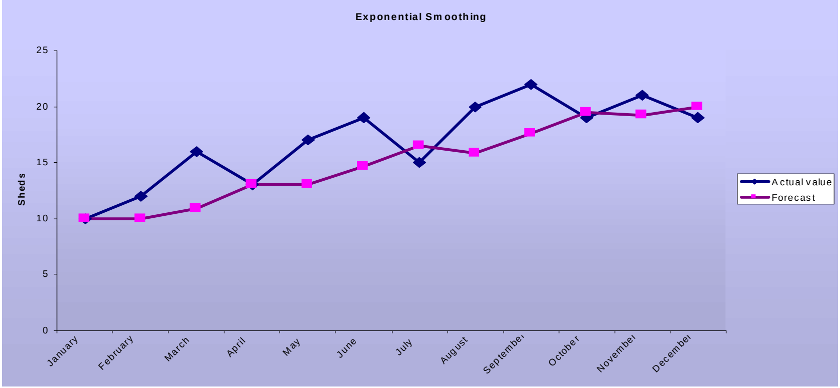
Forecasting Methods - Time Series Models

- Example:

Storage Shed Sales

Period	Actual Value(Y_t)		Y_{t-1}		α		Y_{t-1}		Y_{t-1}		Y_t
January	10	=	10		0.1						
February	12		10	+	0.1	*(	10	-	10	) =	10.000
March	16		10	+	0.1	*(	12	-	10	) =	10.200
April	13		10.2	+	0.1	*(	16	-	10.2	) =	10.780
May	17		10.78	+	0.1	*(	13	-	10.78	) =	11.002
June	19		11.002	+	0.1	*(	17	-	11.002	) =	11.602
July	15		11.602	+	0.1	*(	19	-	11.602	) =	12.342
August	20		12.342	+	0.1	*(	15	-	12.342	) =	12.607
September	22		12.607	+	0.1	*(	20	-	12.607	) =	13.347
October	19		13.347	+	0.1	*(	22	-	13.347	) =	14.212
November	21		14.212	+	0.1	*(	19	-	14.212	) =	14.691
December	19		14.691	+	0.1	*(	21	-	14.691	) =	15.322

Forecasting Methods - Time Series Models



- Example: The table below shows the number of cars sold by Speed Motors in the last 10 days. You need to forecast the sales on day 11 using exponential smoothing

Days	1	2	3	4	5	6	7	8	9	10
Cars Sold	20	13	19	19	25	17	15	13	22	20

- There are two problems with Exponential Smoothing:
  - (a) What value of Alpha to use? Can be found by trial & error.
  - (b) How to get the first forecast? Choose a suitable value or Use warm-up method

Forecasting Methods - Time Series Models

- For this problem we are going to use a warm-up period of 10 days, and therefore our first proper forecast will be for day 11.
- Alpha values of 0.1 and 0.5 will be used for Speed Motors.
- Assume that the forecast for day 2 is the actual for day 1. To calculate the forecast for day 3 we use the formula

Next forecast = alpha*actual demand in the present period + (1-alpha)*previously determined forecast for the present period
$=0.1*13+(1-0.1)*20 = 1.3+18.0 = 19.30$

Days	Actual Sales	Forecast	
1	20		
2	13	20	
3	19		

Forecasting Methods - Time Series Models

- The **simple forecasting** and **smoothing methods** model components in a series that are usually easy to see in a time series plot of the data.
- These methods **decompose** the data into its **trend and seasonal components**, and then extend the estimates of the components into the future to provide forecasts.
- **Static methods** have patterns that do not change over time; **dynamic methods** have patterns that do change over time and estimates are updated using neighboring values.

STATIC (SIMPLE) METHODS

- Trend Analysis
- Decomposition

DYNAMIC (SMOOTHING) METHODS

- Moving Average
- **Single Exponential Smoothing**
- **Double Exponential Smoothing**
- **Winters' Method (Triple Exp. Smoothing)**

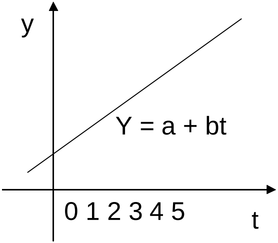
Forecasting Methods - Time Series Models

- **Trend analysis**:
  - technique that fits a trend equation (or curve) to a series of historical data points.
  - Projects the curve into the future for medium and long term forecasts.
  - forecast the future path of economic variables based on historical data
  - use a regression model to model the trend as a function of time
- Types of trend analysis
  - linear trend
  - nonlinear trend

Forecasting Methods - Time Series Models

- Develop an equation that will describe trend: **Linear trend** (simple linear regression model):

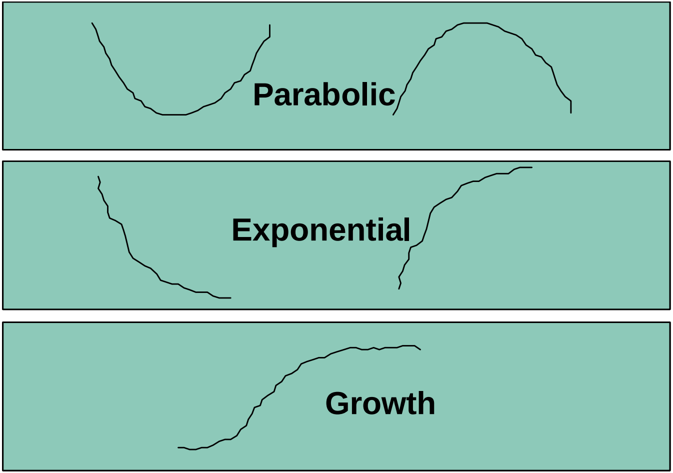
$$Y_t = \beta_0 + \beta_1 t + \varepsilon_t, \; t = 0, 1, 2, ...$$



- β_1 is similar to the slope. However, since it is calculated with the variability of the data in mind, its formulation is not as straight-forward as our usual notion of slope.

Forecasting Methods - Time Series Models

- Common Nonlinear Trends



Forecasting Methods - Time Series Models

- Another possibility is an **exponential trend**, which can be modeled as

$$log(Y_t) = a_0 + a_1 t + \varepsilon_t, t = 1, 2, \ldots$$

- Another possibility is a **quadratic trend**, which can be modeled as

$$Y_t = a_0 + a_1 t + a_2 t^2 + \varepsilon_t, t = 1, 2, \ldots$$

- Adding a linear trend term to a regression is the same thing as using "**detrended**" series in a regression
- Detrending a series involves regressing each variable in the model on t
- The residuals form the detrended series
- An advantage to actually detrending the data (vs. adding a trend) involves the calculation of goodness of fit
  - Time-series regressions tend to have very high R^2, as the trend is well explained
  - The R^2 from a regression on detrended data better reflects how well the x_t's explain y_t

- **Exponential Smoothing with a Trend (Holt's Method: Double Exponential Smoothing)**
- Assumptions: Linear trend and Exponentially declining weights to past observations/trends
- Model:

$$\begin{aligned} F_{t+1} &= \alpha Y_t + (1-\alpha)(F_t + T_t) : \text{ Smooth the base forecast} \\ T_t &= \beta (F_t - F_{t-1}) + (1-\beta) T_{t-1} : \text{ Smooth the trend forecast} \end{aligned}$$

- This time trend is also smoothed, note that previous trend (of t-1) and current trend (of t) appear in the smoothing formula: T_{t-1} and $F_t - F_{t-1}$

Forecasting Methods - Time Series Models

- Ideas behind smoothing with trend:
  - "De-trend" time-series by separating base from trend effects
  - Smooth base in usual manner using α
  - Smooth trend forecasts in usual manner using β
- Forecast k periods into future F_{t+k} with base and trend

$$F_{t+k} = F_t + kT_t$$

Forecasting Methods - Time Series Models

- **Techniques for seasonality**:
  - When a seasonal pattern repeats yearly, this can be used for future forecasts
  - Need monthly or quarterly data
  - A seasonal index is the ratio of the average value in that season, over the annual average
  - Examples: demand for coal in winter months; demand for soft drinks in the summer and over major holidays
  - Seasonality can be dealt with by adding a set of seasonal dummies
  - As with trends, the series can be seasonally adjusted before running the regression
  - Seasonality is expressed as a percentage of the average amount
  - seasonal percentages = seasonal relatives = seasonal indices

Forecasting Methods - Time Series Models

- Seasonal relative = 1.45 for the quantity of television sold in August at Circuit City, meaning that TV sales for that month are 45% above the monthly average.
- Seasonal factor =0.60 for the number of notebooks sold at the UTD bookstore in April, meaning that notebook sales are 40% below the monthly average.
- Deseasonalize historical observations: Divide them by seasonal indices
- Make the analysis = Generate forecasts
- Seasonalize forecasts: Multiply them by seasonal indices

Forecasting Methods - Time Series Models

- **Winter's Method: Exponential Smoothing w/ Trend and Seasonality**
- Ideas behind smoothing with trend and seasonality:
  - "De-trend': and "de-seasonalize"time-series by separating base from trend and seasonality effects
  - Smooth base in usual manner using α
  - Smooth trend forecasts in usual manner using β
  - Smooth seasonality forecasts using γ
- Assume m seasons in a cycle
  - 12 months in a year
  - 4 quarters in a month
  - 3 months in a quarter
  - et cetera

Forecasting Methods - Time Series Models

- Based on 3 equations, each of which smooth a factor associated with one of the three components of the pattern: randomness, trend, and seasonality
- Smooth the base forecast F_t

$$F_t = \alpha \frac{D_t}{S_{t-m}} + (1-\alpha)(F_{t-1} - T_{t-1})$$

- Smooth the trend forecast T_t

$$T_t = \beta (F_t - F_{t-1}) + (1-\beta) T_{t-1}$$

- Smooth the seasonality forecast S_t

$$S_t = \gamma \frac{D_t}{F_t} + (1-\gamma) S_{t-m}$$

Forecasting Methods - Time Series Models

- Forecast F_t with trend and seasonality

$$F_{t+k} = \left(F_{t-1} + kT_{t-1}\right) S_{t+k-m}$$

- Smooth the trend forecast T_t

$$T_t = \beta \left(F_t - F_{t-1}\right) + \left(1 - \beta\right) T_{t-1}$$

- Smooth the seasonality forecast S_t

$$S_t = \gamma \frac{D_t}{F_t} + \left(1 - \gamma\right) S_{t-m}$$