# Análise Matemática II - Formulário

$$z = x + iy = r(\cos\theta + i\sin\theta)$$
$$r = |z| = \sqrt{x^2+y^2},\ \theta = \arg z$$

$$x = \frac{1}{2}(z+\bar{z}) \quad e \quad y = \frac{1}{2i}(z-\bar{z})$$

$$f(z) = u(x,y) + i\, v(x,y)$$

**Cond. Cauchy-Riemann**

$$\begin{cases} \dfrac{\partial u}{\partial x}(x_0,y_0) = \dfrac{\partial v}{\partial y}(x_0,y_0) \\ \dfrac{\partial u}{\partial y}(x_0,y_0) = -\dfrac{\partial v}{\partial x}(x_0,y_0) \end{cases}$$

$$f'(z) = \frac{\partial u}{\partial x}(x_0,y_0) + i\frac{\partial v}{\partial x}(x_0,y_0)$$

$$e^z = e^x e^{iy} = e^x(\cos y + i\sin y)$$

$$\sin z = \frac{e^{zi}-e^{-zi}}{2i} \quad e \quad \cos z = \frac{e^{zi}+e^{-zi}}{2}$$
$$\sinh z = \frac{e^{z}-e^{-z}}{2} \quad e \quad \cosh z = \frac{e^{z}+e^{-z}}{2}$$
$$\log z = \ln|z| + i\arg z, \arg z \in [-\pi,\pi[$$

$$(e^z)' = e^z \qquad (\log z)' = \tfrac{1}{z}$$
$$(\sin z)' = \cos z \qquad (\cos z)' = -\sin z$$
$$(\sinh z)' = \cosh z \qquad (\cosh z)' = \sinh z$$
$$(f(z)\,g(z))' = f'(z)\,g(z) + f(z)\,g'(z)$$
$$\left(\frac{f(z)}{g(z)}\right)' = \frac{f'(z)\,g(z) - f(z)\,g'(z)}{g^2(z)}$$

**Integral Curvilíneo** ao longo de $\gamma$

$$\int_\gamma f(z)\,dz = \int_a^b f(r(t))\ r'(t)\ dt$$

$r(t)$ parametriz. de $\gamma$, $a \le t \le b$

**Fórmula Integral de Cauchy**

$$\int_\gamma \frac{f(z)}{z-z_o}dz = 2\pi i\ f(z_o)$$

**Fórmula Integral de Cauchy (derivadas)**

$$\int_\gamma \frac{f(z)}{(z-z_o)^{n+1}}dz = \frac{2\pi i}{n!}\ f^{(n)}(z_o)$$

**Teorema dos Resíduos**

$$\oint_\gamma f(z)dz = 2\pi i \sum_{k=1}^{n} \text{Res } f(z_k) \quad \text{onde}$$

$$\text{Res } f(z_0) = \frac{1}{(m-1)!} \lim_{z\to z_0} \frac{d^{m-1}}{dz^{m-1}}[(z-z_0)^m f(z)]$$

sendo $z_0$ um polo de ordem $m$ de $f$

Segmento de recta entre dois pontos $A$ e $B$ : $r(t) = A + t(B-A), a \le t \le b$

Parametrização de uma circunferência: $|z| = a;\ r(t) = a\cos t + a\sin t = ae^{it},\ 0 \le t \le 2\pi$

**Integrais de Linha**

de um campo escalar $f$, $\int_\gamma f\ ds = \int_a^b f(\overrightarrow{r}(t))\,\|\overrightarrow{r}'(t)\|\ dt$

de um campo vectorial $\overrightarrow{F}$; $\int_C \overrightarrow{F}\left|\overrightarrow{dr}\right. = \int_a^b \overrightarrow{F}(\overrightarrow{r}(t))\,|\overrightarrow{r}'(t)\ dt$

**Teorema de Green**

$$\overrightarrow{F}(x,y) = (P(x,y), Q(x,y))\,; \quad \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy = \oint_\gamma \overrightarrow{F}\left|\overrightarrow{dr}\right.$$

Definição do limite de $f(x,y)$ num ponto $(a,b)$

$$\forall \delta > 0, \exists\, \varepsilon > 0 : \sqrt{(x-a)^2 + (y-b)^2} < \varepsilon \ \wedge \ (x,y) \in D_f \setminus \{(a,b)\} \Longrightarrow \ |f(x,y) - l| < \delta$$

Diferenciabilidade de $f(x,y)$ num ponto $(a,b)$

$$f(a+h,b+k) - f(a,b) = h\left(\frac{\partial f}{\partial x}\right)_{(a,b)} + k\left(\frac{\partial f}{\partial y}\right)_{(a,b)} + \varepsilon(h,k)\sqrt{h^2+k^2}, \text{ onde } \lim_{\substack{h\to 0\\ k\to 0}} \varepsilon(h,k) = 0$$

Derivadas parciais de primeira ordem

$$\left(\frac{\partial f}{\partial x}\right)_{(a,b)} = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h} \quad \text{e} \quad \left(\frac{\partial f}{\partial y}\right)_{(a,b)} = \lim_{k\to 0} \frac{f(a,b+k) - f(a,b)}{k}$$

Derivada direccional de $f(x,y)$ no ponto $(a,b)$ segundo o vector $\overrightarrow{u} = (u_1, u_2)$

$$f'_{\overrightarrow{u}}(a,b) = \left(\frac{\partial f}{\partial x}\right)_{(a,b)} u_1 + \left(\frac{\partial f}{\partial y}\right)_{(a,b)} u_2$$

Operadores diferenciais

$$\text{Para } \vec{F}(x,y,z) = \left(F_x(x,y,z), F_y(x,y,z), F_z(x,y,z)\right) \text{ temos}$$

$$\text{div } \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\overrightarrow{\text{rot } \vec{F}} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$