The Statistical Foundations of Ratio Analysis: Theory and Hong Kong Evidence

By

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To my parents

The age of chivalry has gone:
that of sophisters, economists and accountants has succeeded.
(E. Burke)

Preface

This study proposes a theoretical framework for understanding the statistical characteristics of financial ratios while presenting compelling evidence on the multiplicative character of these variables. It is divided into two parts, one where ratio components (and the ratios themselves) are studied using mostly a cross-section viewpoint (many different firms taken at the same moment in time) and the second part where ratios are studied using mostly a stochastic viewpoint. However, this separation is not taken too far as it would become rather artificial.

The contents of the study are of two types: theoretical and empirical. In some occasions there is no clear separation between these two types. The data used to test hypotheses comes from a database of Hong Kong firms between the years of 1979 to 1991. We regret that the small period available precludes a satisfactory description and hypotheses testing in the case of stochastic processes.

Scientific Background

Accounting academics have often considered the widespread use of ratios in financial analysis as a somewhat intriguing practice. Indeed, in one of the earliest contributions to this topic in the accounting research literature, Horrigan (1965) remarked that financial ratios were referred to in text books in almost apologetic tones as though their expected utility were extremely low. Horrigan's response to this situation, however, was to seek to dissipate such doubts by describing the statistical characteristics of some widely used ratios in order to demonstrate that they may be useful after all.

Following Horrigan's optimistic review, subsequent empirical research revealed some promising applications of financial ratio analysis. Beaver (1966) and Altman (1968), for example, showed that ratios have the potential to predict the bankruptcy of firms. A few years later, however, the skepticism returned, with authors such as Deakin (1976) noticing that the empirical frequency distributions of financial ratios appear to vary widely and, as a result, questioning the validity of analytical methods which assumed the normality of ratio data. This prompted Frecka & Hopwood (1983) and others to propose *ad hoc* techniques such as transformation and the trimming or winsorising of outliers to deal with the unduly influential values present in samples of financial ratios, techniques which reflected the apparently widespread belief that there were no general rules underpinning the ratio method.

Adding to the growing doubt concerning the validity of financial ratio analysis, Lev & Sunder (1979) raised some fundamental questions as to whether the use of ratios is motivated by well-founded considerations or whether, in contrast, it is merely a tradition. These authors claimed that almost all of the assumptions required for valid ratio analysis are likely to be violated in practice. A more balanced critique by Whittington (1980) followed, not only uncovering cases where ratios seem not to be up to the task but also distinguishing the cross-sectional (or inter-firm) application of ratios from their less-acceptable use in prediction.

Both Lev & Sunder (1979) and Whittington (1980) stressed that valid measurement using ratios requires proportionality between the components (i.e. $Y = bX$). Since such an assumption seems to be too restrictive, these authors advocated a two parameter regression model ($Y = a + bX$), or similar functional form, rather than the single parameter ratio model. Barnes (1982) went one step further, suggesting that the difficulties caused by skewed distributions in ratios also stem from non-proportionality. Barnes showed that the use of regressions or similar functional forms instead of ratios should eliminate both the problem of non-proportionality and that of skewness in distributions.

A striking feature of the above-mentioned research is the small impact it has had, both on the practice of financial analysis and in the way empirical research is carried out. One reason for this may be that researchers pre-suppose that accounting data is a special case, too complex for simple, unifying explanations. As a consequence, empirical research based on accounting data lacks the level of definition that is required to make the specific assumptions that are needed to draw appropriate inferences. For example, in spite of the insistence that financial ratios are constructed in order to remove the effect of firm size, (thus making it possible to compare firms of different sizes,) to date the literature on ratio analysis has not produced a definition of 'size' itself. If the aim of a ratio is indeed to remove size, no progress can be made until specific definitions of firm size are tested against the assumptions of the ratio model.

Description of Purpose and Contents

In an earlier attempt to base ratio models on broader assumptions, it was demonstrated that a fuller understanding of financial ratios can be achieved by taking account of the behaviour of the variables from which financial ratios are constructed, particularly where firm size plays an important part (McLeay, 1986*a*). At the same time, some limiting case theoretical ratio models which allow for exponential growth in accounting variables were identified (McLeay, 1986*b*). Subsequently, Tippett (1990) proposed an inductive methodology for modelling financial ratios where the components are again assumed to be multiplicative, although no assumptions are made about firm size. The distributional properties of such ratios are documented by Rhys & Tippett (1993).

The thesis is inspired by McLeay or Tippett rather than by Barnes or Frecka & Hopwood. It is an attempt to demonstrate that financial ratios obey clear distributional rules and follow well-defined types of processes. To this effect, Chapter 1 explains why the distributions of ratios seem to vary widely, showing that, underlying its variety, is a set of rules easy to understand. Chapter 2 then empirically tests these theoretical insights, presenting compelling evidence showing that such rules are verified, in practice, in Hong Kong data.

In the second part of the thesis, evidence is also provided on the time-series characteristics of accounting data in Hong Kong. The last two chapters of the thesis are again theoretical, introducing to the readers the essence of Stochastic Calculus and showing, based on such introduction, that the validity of financial ratios is possible even where their components are diffusions.

Acknowledgements

This study would never be accomplished without the contribution of any of the following people. First of all, I would like to give my sincere gratitude to my supervisor, Prof. Duarte Trigueiros, for his enlightening guidance, enormous support and infinite patience throughout the preparation of the thesis. Out of his tight schedule of teaching and research, Prof. Trigueiros always manages to discuss my work and give me his inspiring comments. I deeply acknowledge that most of the theorectical developments presented in this study are the direct result of such supervision, which enabled me to follow and improve upon those developments and to apply them to the case of Hong Kong data.

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Contents

Part I

Cross-Section Theory and Evidence

Chapter 1

The Distribution of Financial Ratios: Theory

Accounting reports are an important source of information for managers, investors and financial analysts. Ratios are the usual instruments for extracting this information. However, statistical characteristics of ratios pose particular difficulties when used because their distributions vary widely: most ratios are positively skewed but some are symmetrical and others are negatively skewed. Despite the widespread use of ratios in a multitude of contexts, available evidence probably conveys the belief that the distribution of ratios is unpredictable. In fact, researchers and practitioners alike still rely on *ad hoc* transformations and outlier deletion in order to adjust the distribution of ratios to approximate normality.

The first part of this thesis explains the influence of accounting identities on the statistical distribution of financial ratios. First, this study recalls that, where accounting numbers are lognormally distributed, then ratios are expected to be positively skewed. Accordingly, the fact requiring an explanation is why some ratios are symmetrical or even negatively skewed, not why the distribution of ratios is positively skewed. The study then shows that apparently symmetrical ratios occur because accounting identities act as external boundaries, constraining the long tail of their otherwise skewed distribution to become much smaller. Ratios that are symmetrical or negatively skewed simply reflect the existence of these boundaries. They revert to positive skewness after being inverted, thus making it difficult to accept the hypothesis that the skewness of ratios stems from other causes such as non-proportionality. Since bounded ratios may induce misleading results when used for calculating confidence intervals or P values, a procedure is suggested to avoid constraints where necessary.

The specific purpose of the present chapter is to provide the theoretical explanation for the

TA	Total Assets	NW	Net Worth
FA	Fixed Assets	DB	Long Term Debt
D	Debtors	C	Creditors
CA	Current Assets	CL	Current Liabilities
I	Inventory	TC	Total Capital Employed
WC	Working Capital	TD	Total Debt
EX	Operating Expenses less Wages	S	Sales
OP	Operating Profit	QA	Quick Assets

Table 1: List of abbreviations used in this study.

existence of symmetrical and negatively skewed ratios. Our explanation is then supported by evidence on cross-section distributions of accounting numbers and ratios, using data extracted from accounting reports of large Hong Kong firms. This evidence will be presented in Chapter 2.

Table 1 shows the accounting identities and the abbreviations used in both chapters.

1.1 Literature Review

The statistical distribution of ratios has been the object of considerable study. Horrigan (1965), in an early work on this subject, reported positive skewness in some ratios and explained it as a result of effective lower limits of zero. O'Connor (1973) and Bird & McHugh (1977) also found skewness in ratios. Deakin (1976) showed that, in most ratios, positive skewness could not be ignored but also noticed that the ratio TD/TA was near normality. Bougen & Drury (1980) reported skewness, either negative or positive, and extreme outliers. Frecka & Hopwood (1983) extended Deakin's study and reported similar findings. These authors proposed applying transformations and then trimming or winsorising outliers as a means of reaching normality. Ezzamel, Mar-Molinero & Beecher (1987) noticed positive skewness and outliers except for ratios TD/TA and NW/TA and found improvements with square root and logarithmic transformations. So (1987) also found positive skewness except in ratios TD/TA, NW/TA and CA/TA, the latter being negatively skewed. Watson (1990) and Karels & Prakash (1987) studied the multivariate normality of ratios and the advantage of removing multivariate outliers. They noticed that ratios TD/TA and NW/TA were near normality. The same was observed by Ezzamel & Mar-Molinero (1990) who suggested that the trimming of 'obvious' outliers should come first, instead of transforming and then trimming, as proposed by Frecka & Hopwood.

McLeay (1986*a*; 1986*b*) questioned the use by some researchers of such ad-hoc procedures as transformation and trimming of remaining outliers as a means of achieving normality in ratios. He suggested that the data should be left unadjusted and better-fitting models should be used. Tippett (1990) and Rhys & Tippett (1993) developed stochastic processes aimed at identifying the distributional characteristics of ratios.

1.2 Multiplicative vs Additive Data

Authors mentioning cross-section lognormality in accounting aggregates explain it as the outcome of multiplicative processes such as the geometric brownian motion (Tippett, 1990). These processes are considered plausible where variables are the result of accumulations, that is, where they are sums of similar transactions with the same sign (McLeay, 1986*a*).

The theory we propose here is based upon similar assumption. However, there is a difference in emphasis. While the authors mentioned above stress the importance of generative mechanisms underlying every item, we focus on the overall effect of size. Instead of assuming that only accumulations such as Sales and Stocks are lognormal, we accept that Sales, Stocks and other aggregates are expected to be lognormal because the growth of the firm as a whole is a stochastic accumulation. Since the effect of size in accounting aggregates cannot be discarded on *a-priori* grounds, we are inclined to see lognormality as the rule rather than as the exception. The evidence presented later in this paper supports this view.

Where accounting aggregates are lognormally distributed, then the logarithm of an observation x_j from financial report j is explained as the expected value of the transformed variable, μ, plus a deviation or residual, e_j. An estimated μ is $\overline{\log x}$, the mean of $\log x$. Ratios y/x can be written as a difference of logarithms:

$$\log y_j - \log x_j = (\mu_y - \mu_x) + (e_y - e_x)_j \quad \text{corresponding to} \quad \frac{y_j}{x_j} = Rf_j \tag{1}$$

where R is an expected proportion and an estimated R is given by $\exp(\overline{\log y} - \overline{\log x})$, the median of the ratio. Therefore, f_j is, for report j, the percent deviation from the median of the ratio. On a logarithmic scale, this deviation is a difference, $(e_y - e_x)_j$ which we refer to in this thesis as $\varepsilon_j^{y/x}$. The distribution of the ratio y/x is the same as the distribution of $f = \exp \varepsilon^{y/x}$. Given that f is an exponentiation of $\varepsilon^{y/x}$, then ratios are expected to be lognormal (McLeay, 1986*a*).

When considering the statistical distributions of ratios, most authors have assumed that the difficulties posed by their atypical behaviour are caused by distortions of normality. However, the peculiar characteristics of lognormal variables and the consequences ensuing from their use must be borne in mind in any context involving the statistical manipulation of ratios. Lognormality cannot be treated as a simple distortion of normality. No distorting mechanism would be able to create, from additive events, the wide range of values generally found in multiplicative variables. For instance, the larger values observed in a lognormal sample are likely to be many hundreds of times larger than the smaller ones. Such extreme proportions have no counterpart in those observed in additive variables. Indeed, as shown in figure 1.2, most of the realisations of the lognormal distribution are concentrated in a small region, while larger values are spread out over a wide range.

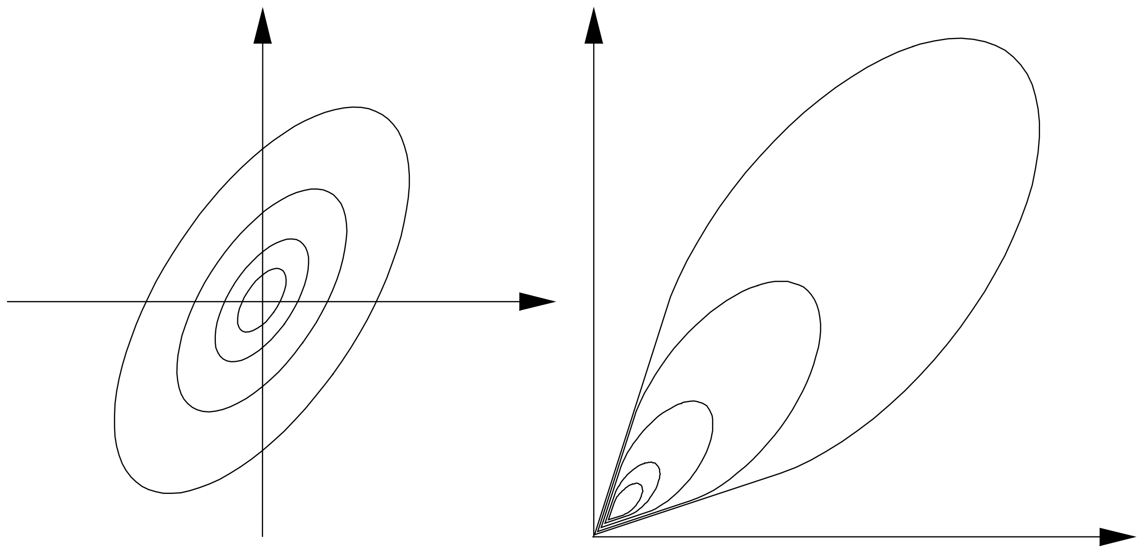

Figure 1: Two stylised representations of bivariate distributions. On the left, the density contours of a typically additive distribution; on the right, the corresponding multiplicative distribution.

Of particular interest is the fact that lognormal distributions are very skewed, exhibiting long tails towards positive values. For coefficients of variation[1] beyond 0.25, most of the observations concentrate in a small region with only a few extreme values spreading out over a wide range. It is easy to interpret these extreme values as outliers (Snedecor & Cochran, 1965, p. 281; McLeay, 1986*b*, p. 209; Ezzamel & Mar-Molinero, 1990, p. 13). In fact, outliers often mentioned in relation to ratios are probably just a consequence of multiplicative skewness.

Moreover, accounting variables (like other economic phenomena such as income, wealth, stock prices and the size of firms) cannot be described as resulting from the kind of additive stochastic process which underlies normal variables. Indeed, accounting realisations are generated by a multiplicative rather than an additive law of probabilities. That is, whilst each transaction contributing to the amount reported as, say, Total Sales for a given period is itself a random event, an individual transaction contributes to the reported aggregate not in a manner which could lead to either an increase or decrease in Total Sales, but by accumulation only. Accumulations of random events tend to be multiplicative, as opposed to additive, because the likelihood of realisations is conditional on the occurrence of a chain of several previous events. Such likelihood thus stems from multiplying, rather than adding probabilities.

When the multiplicative character of accounting data is ignored, features of the data which would otherwise be considered as commonplace (such as extreme values) are likely to be seen as extraordinary. Given that such observations are not unusual in financial ratio samples, the technique of trimming or outlier deletion advocated by authors such as Frecka & Hopwood (1983) is a questionable practice. In fact, since the mechanism commanding the emergence of extreme values holds when the scale differs, and as trimming is in some respects equivalent to a reduction in scale, it will be found that the exclusion of one extreme value merely leads to its replacement by another extreme value on the reduced scale.

1.3 The Random Effect of Size

A further reason why it is important to assume the correct type of statistical behaviour is that adequate descriptions of the inter-relationships amongst variables differ between additive and multiplicative variables. For additive data, it is assumed implicitly that distributions are preserved when variables are added or subtracted. This is not the case for multiplicative data where distributions are preserved when variables are multiplied or divided.

For example, the simplest additive formulation is $x_i = \mu + z_i$, where x_i is explained as an expected value, μ, plus a random deviation, z_i, specific to case i. The multiplicative equivalent would be $x_i = x_0 f_i$, where the i^{th} realisation of x is explained as an expected magnitude, x_0, which is the same in all cases, multiplied by a random proportion of x_0, f_i, specific to i.

When an additive variable x is explained not only by an expected value, μ, but also by d, an extra component of the variance of x, then $x_{ji} = \mu + d_j + z_i$ where d_j is the expected deviation from μ introduced by the j^{th} level of d. If the same d_j is present in two variables, y and x, it is possible to remove it from measurement by subtracting variables. For instance, when a medical trial is carried out in the same group of patients both before and after treatment, the difference between observations y and x measures the effect of the treatment and is free from spurious influences such as those of the sex of the patient because such factors, being present in both observations, cancel out when subtracted.

The ratio method may be viewed as the multiplicative equivalent of the above example, where firm size is the 'spurious' influence to be removed from the ratio components y, x. In fact, the simplest random generalisation of a deterministic growth process to allow for randomness would lead to the definition of accounting variables y, x, as reported in the j^{th} financial statement, as

$$y_{ji} = y_0 e^{s\tau_j} f_{y_i} \qquad \text{and} \qquad x_{ji} = x_0 e^{s\tau_j} f_{x_i}. \tag{2}$$

y_0, x_0 are the expected magnitudes of y, x and the common term $e^{s\tau_j}$ plays the same role as d_j above: it is the component of the variance of y, x present in both observations. f_{y_i}, f_{x_i} are random proportions of y, x unexplained by the model.

In the above *random effects*[2] models, y_0, x_0 are specific to variables y, x (they are independent of the particular financial statement under analysis), whereas $e^{s\tau_j}$ (the number of times any item in the j^{th} financial statement is likely to be larger or smaller than the expectation) scales all items in the financial statement irrespective of changes in the particular variables of interest. It is this

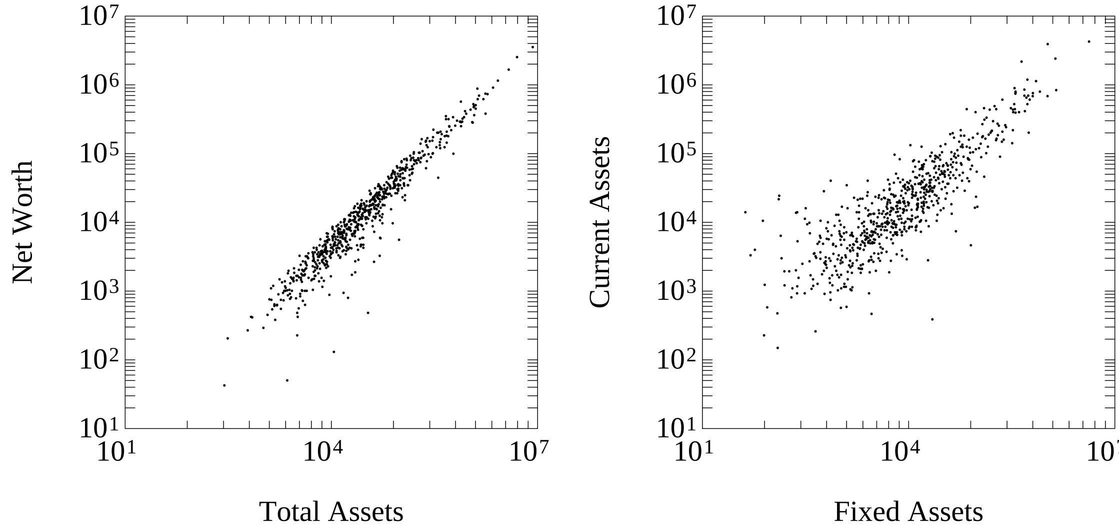

Figure 2: Two scatterplots comparing constrained (left) with unconstrained (right) bivariate distributions of raw numbers. Each dot is one firm. The axes use logarithmic scaling. The constraining frontier is the bisecting line $\log x = \log y$.

separation between effects which ensures that the ratio y/x removes $e^{s\tau_j}$, the effect attributable to the j^{th} financial statement, thus making company accounts comparable either through time short periods of or across firms.

Notice that, formulations such as (2), where both the expectation and the variance are constant, may be a good approximation to model the behaviour of accounting variables in cross-section. However, in a time-series context, (2) is far from adequate as the mean and variance of accounting variables are non-stable over time. This issue will be addressed in the second part of the study.

1.4 The Distribution of Bounded Ratios

If components of ratios are lognormal, then ratios should be positively skewed. Although most ratios exhibit positive skewness, several authors also mention ratios which are symmetrical or even negatively skewed. As mentioned above, TD/TA and NW/TA have been reported as being Normal and CA/TA has been found to be negatively skewed. How is this possible? The reason seems to be straightforward. Accounting identities make it impossible for some ratios to take on all the values a skewed distribution allows. This constraint is clearly observable when plotting, on a logarithmic scale, the two components of a ratio against each other. Figure 2 compares a constrained ratio with an unconstrained one. The figure shows, on the left, the effect of a boundary imposed by Total Assets on the spread of Net Worth (where the observed values are scattered below the 45^o bisecting line) and, on the right, an unconstrained relationship.

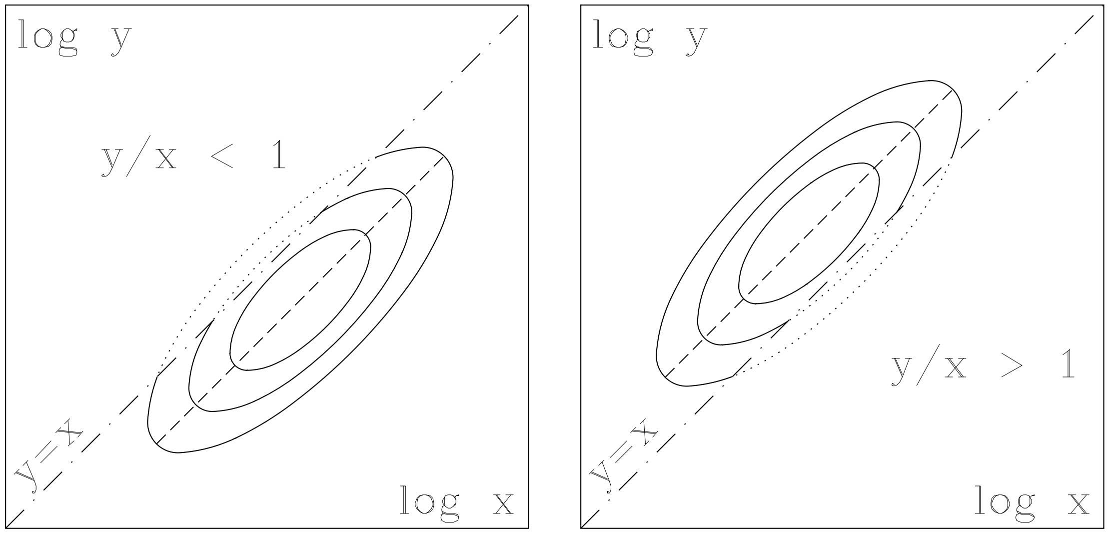

Figure 3: The two kinds of constraints affecting bivariate distributions of raw numbers. The axes use logarithmic scaling.

There is a constraint if, due to an accounting identity or other external cause, the ratio relationship $y_j/x_j = R \times f_j$ is bounded by one of the following inequalities:

$$\text{for any } j, \quad x_j > y_j \quad \text{or} \quad y_j > x_j$$

The inequality on the left can be found in ratios in which the numerator is bounded by the denominator such as Current Assets to Total Assets. The inequality on the right arises in ratios in which the denominator is bounded by the numerator (it is possible to create the latter by taking the inverse or *reciprocal* of the former).

Figure 3 illustrates the two types of constraint. Where the constraint is $x_j > y_j$, ratios cannot be larger than 1. The effect of this constraint on the distribution of ratios is that it inhibits the spread of its otherwise positively skewed distribution. Instead of the large, lognormal-like tail to the right, such ratios exhibit a smaller one. Where the constraint is $y_j > x_j$, ratio values cannot be lower than 1. The large, lognormal-like tail is unaffected, but the left hand tail is truncated, thus increasing even more the positive skewness of the ratio. In both cases, the bisecting line $x = y$ (or $\log x = \log y$) acts as a boundary. Accordingly, positive skewness would emerge after inverting one of the apparently Normal ratios.

It should be possible to broadly predict the decrease in skewness introduced by a given boundary. Where the constraint is $x_j > y_j$, then $\overline{\log y} - \overline{\log x} < 0$ and $\varepsilon_j^{y/x} < -(\overline{\log y} - \overline{\log x})$ for any j. That is, large positive deviations from the expected value are not allowed. Where the constraint is $y_j > x_j$, then $\overline{\log y} - \overline{\log x} > 0$ and $\varepsilon_j^{y/x} > -(\overline{\log y} - \overline{\log x})$ for any j. That is, large negative deviations from the expected value are not allowed. Since, in both cases, a constraint prevents $\varepsilon^{y/x}$ from being larger than $\overline{\log y} - \overline{\log x}$, then the nearer $\overline{\log x}$ is to $\overline{\log y}$, the stronger the constraint. Thus the difference $\overline{\log y} - \overline{\log x}$ can be used to estimate the extent to which constraints affect the symmetry of the distribution of $\varepsilon^{y/x}$. Taking the spread of $\varepsilon^{y/x}$ into account we obtain the normalized difference

$$\zeta = \frac{\overline{\log y} - \overline{\log x}}{\sqrt{\mathrm{VAR}(\varepsilon^{y/x})}}. \tag{3}$$

In standard deviation units, $|\zeta|$ is the distance separating the constraining boundary from the expected value of the log-ratio. For $|\zeta| > 2$, the constraint is small (less than 2.5% of firms are expected to have their ratios constrained). Thus the lognormal tail or skewness is almost unaffected. For $2 > |\zeta| > 1$, the constraint becomes significant, causing symmetrical or even negatively skewed ratios, as more than 16% of firms are expected to have their ratios constrained.

Besides accounting identities, there are other external factors which may affect the distribution of ratios. However, instead of defining boundaries which are impossible to cross, they bring about a decrease in the density of observations. For example, as firms are likely to avoid negative Working Capital, the inequality $CA > CL$ will influence the density of the distribution of the Current ratio.

1.5 Avoiding Constraints and Negative Cases

Where the numerator of a ratio is bounded by the denominator, then a simple transformation can take into account the underlying inequality, yielding a new, unbounded ratio. In fact, for any proportion written as

$$\frac{x_i}{\sum x_i}$$

it is possible to calculate the corresponding 'odds ratio', defined as

$$\frac{x_i}{(\sum x_i) - x_i}.$$

For example, the odds ratio corresponding to FA/TA is the ratio FA/CA as $CA = TA - FA$. The information contained in both ratios is the same. The difference between odds-like ratios and the corresponding proportion-like ones is just functional. It is therefore possible to avoid ratios affected by constraints by using the corresponding odds ratios instead.

Now, by considering other transformations and their interaction with accounting identities, it is possible to transform other types and bounded ratios into lognormality. In fact, many types of financial ratio may be described by transforming a simple function Y/X. Indeed, as accounting variables are governed by relatively simple accounting identities, most commonly-used financial ratios can be expressed by rearranging Y and X. For instance, as Earnings may be represented as $X - Y$, i.e. Sales less Total Costs, then Return on Sales is equivalent to $(X - Y)/X$, or $1 - Y/X$. Similarly, where Total Assets is identical to the sum of Equity Capital and Total Liabilities, the Liability Ratio is equivalent to $X/(Y + X)$, or $1/(1 + Y/X)$. Table 1 shows how the meaningful rearrangement of Y

Ratio	f(Y/X)	Boundaries
Y/X	Y/X	$0, \infty$
(X−Y)/X	1−Y/X	$-\infty, 1$
(Y−X)/X	Y/X−1	$-1, \infty$
(Y+X)/X	1+Y/X	$1, \infty$
X/(Y+X)	$(1+Y/X)^{-1}$	0, 1

Table 2: Classes of Financial Ratio.

and X leads to a set of ratios encompassing the classes most commonly observed in practice, each a function of Y/X.

When no accounting identity links ratio components to each other (as, for example, is the case of the Return on Investment ratio) transformations outlined above do not apply. Negative and positive ratios should be viewed as forming, two different models. Considering, for instance, random values, such ratios should be viewed as having, not one unique distribution, but rather a juxtaposition of two distributions, one for positive and the other for negative realisations.

Figure 4 shows that the relationship between size and an accounting aggregate which permits both negative and positive values requires two slopes (i.e. two ratios) each with its origin at zero, one for positive realisations (A) and another for negative realisations (-A). If a single slope were taken to describe the relationship across firms of different size, the implicit assumption would be either that firm size is allowed to be negative (which is impossible) or else, if size must be positive, that variables such as Earnings or Working Capital where negative numbers occur must be independent of size. The only line in figure 4 which is size-independent is the vertical line in the first plot, but since realisations of accounting variables, either positive or negative, are size-dependent (otherwise ratios would not be necessary anyway), this must contradict the facts.

Practitioners, when assessing industry norms for example, tend to disregard some negative numbers. However, in other cases (e.g. in the building of Z-scores), the use of a unique ratio seems more appealing. Anyway, even where the assumption of a single ratio with a distribution that is common to both negative and positive values may look appealing, such a ratio will not remove the effects of size.

Figure 5 provides a representation of the distributional characteristics of variables which can take on negative values, with both positive and negative numbers being proportional to size. It is now clear why such distributions should be viewed as a juxtaposition of two multiplicative distributions.

1.6 Notes

1. The coefficient of variation is the standard deviation expressed as a fraction of the expected value. It is preferable to the standard deviation or the variance for quantifying the spread of

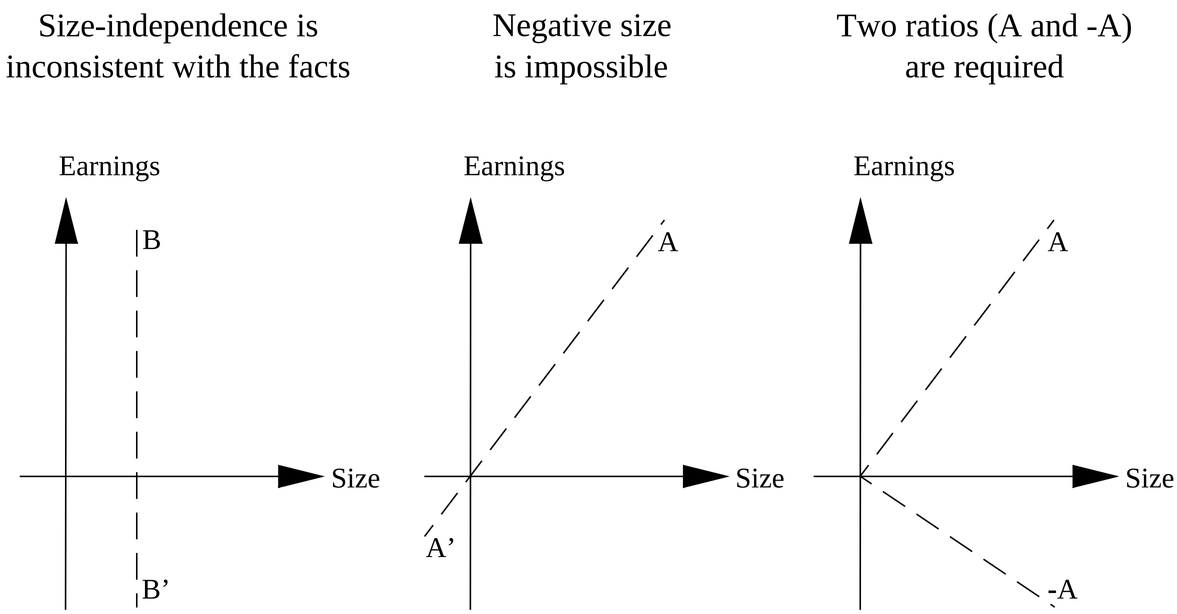

Figure 4: Modelling the relationship between Earnings and Size requires two ratios (slopes A and -A) for positive and negative Earnings respectively.

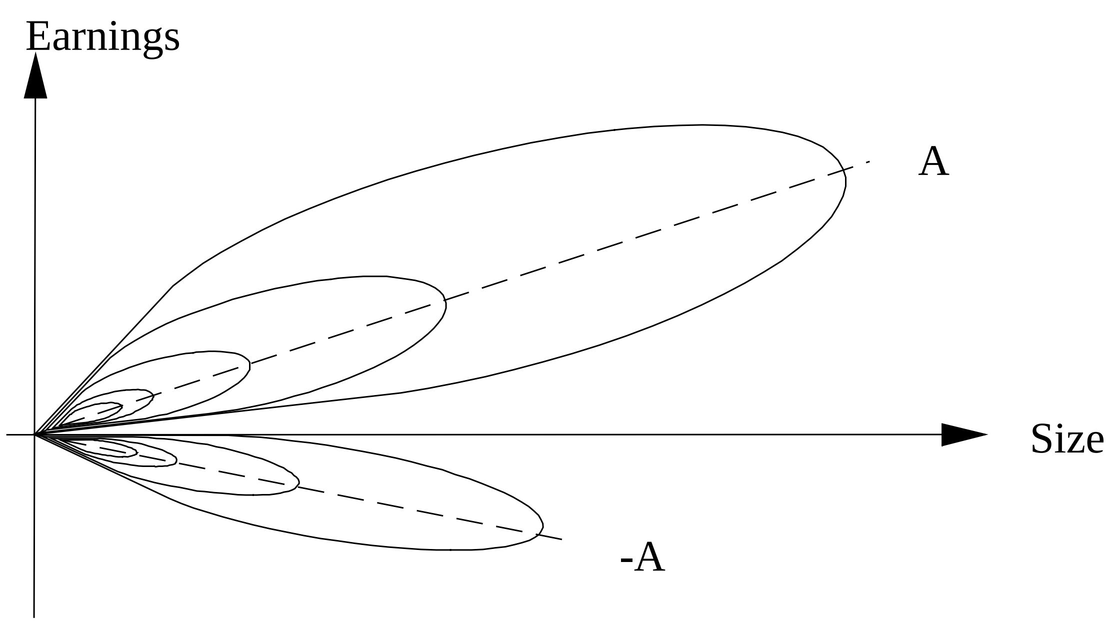

Figure 5: Contour plot representing the density of multiplicative bivariate distributions where one of the variables may take on negative values.

lognormal data, because the latter are not constant.

2. Effects model differences in relation to the expectation, where such differences are introduced by components of the variance. Influences such as the two possible sexes are referred to as 'fixed effects' because the component introduced by the difference in sex is deterministic. Where the component is itself a random variable, the effects are called 'random effects'. Size is therefore a random effect. The different financial statements present in a sample represent the levels of an effect. In statistical terms, the accounting numbers in the same financial statement are said to belong to the same level.

Chapter 2

Evidence on Lognormality of Accounting Aggregates and Ratios

This chapter provides extensive evidence on the cross-section lognormality of aggregates and their ratios extracted from the accounting records of Hong Kong firms. Lognormality in items such as Sales, Earnings and Total Assets has received a great deal of attention in texts on the theory of the growth of firms.[1] Since those texts were not oriented towards the analysis of financial statements, they omitted items which are frequently employed as components of ratios, thus failing to supply the kind of evidence required for building the statistical basis of ratio analysis.

McLeay (1986*a*), in one of the few studies contemplating distributions of aggregates as opposed to ratios, argues that items such as Sales, Stocks, Creditors or Current Assets are expected to exhibit cross-section lognormality. Our empirical work confirms this and suggests that the phenomenon of lognormality is much more widespread. Many other positive-valued items have cross-section distributions that are lognormal. Furthermore, where items can take on positive and negative values, then lognormality can be observed in the subset of positive values and also in the absolute values of the negative subset. Our empirical work has also uncovered cases of three-parametric lognormality. Three-parametric lognormality is important for elucidating the origins of non-proportionality in the relationship between the numerator and the denominator of a financial ratio.

2.1 Methodology and Data Set

The lognormality of items was tested by applying two- or three-parametric logarithmic transformations where appropriate. While the Normal distribution is completely specified by the mean and standard deviation, the lognormal distribution may require one extra parameter in order to account for overall displacement of the distribution. Where a displacement of item x (say $x - \delta$),

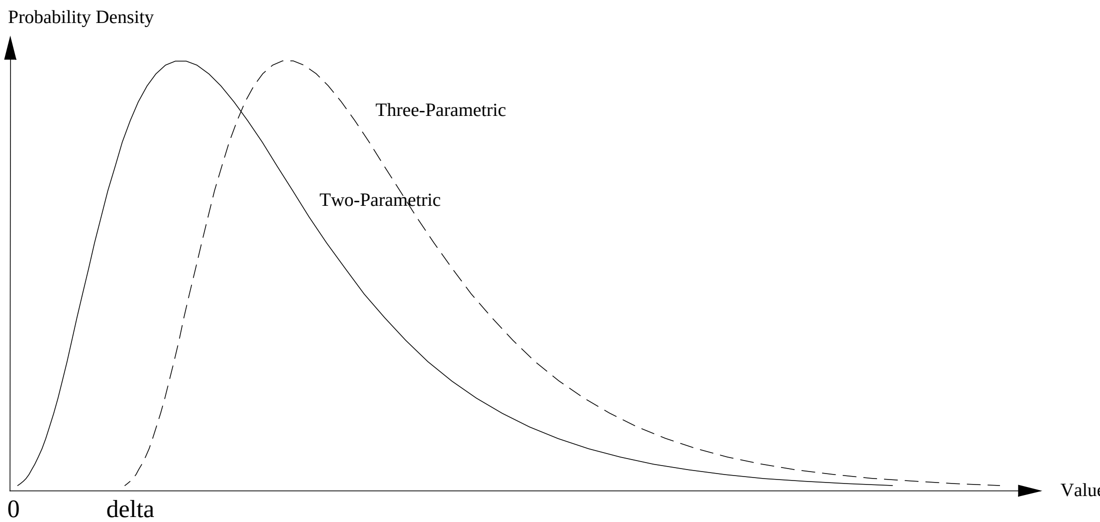

Figure 6: The usual, two-parametric, lognormal distribution (solid line) and the corresponding three-parametric distribution (dashed line) showing a positive displacement, *delta*, known as the threshold.

and not x itself, is Normal after a logarithmic transformation, the distribution of x is known as *Three-Parametric* Lognormal. The range of x is thus $\delta < x < \infty$. The usual, *Two-Parametric*, lognormal distribution is a special case for which $\delta = 0$. Since δ is a lower bound for x, it is known as the *threshold* of the distribution (Aitchison & Brown, 1957). The existence of Fixed Costs, for instance, leads to a constant displacement in the distribution of Total Costs.

Adequate removal of size requires, in this case, one of the following ratios

$$\frac{y - \delta_y}{x} = Pf_{y/x} \tag{4}$$

$$\frac{y}{x - \delta_x} = Pf_{y/x} \tag{5}$$

for, respectively, three-parametric lognormal numerators and denominators. Thresholds are δ_y in (4) and δ_x in (5). The above 'threshold ratios' cope with an elementary form of non-proportionality albeit obeying the same postulates as traditional ratios. Where proportionality holds, then $\delta = 0$ and they revert to the traditional ratio. If thresholds in the numerator and in the denominator are both significant, a reinforcement of non-proportionality occurs when δ_y and δ_x have different signs.

The normality of the transformed observations was assessed using an improved version of the Shapiro-Wilk test (Royston, 1982).[2] This test can cope with large or small sample sizes and is recommended as a superior *omnibus* test. Notice that the subtraction of δ from x is not similar to the practice of adding a constant value to observations for avoiding negative values (Ezzamel & Mar-Molinero, 1990) as the subtraction of δ never changes the sign of observations.

The data set used in this study was taken from the PACAP database for eight consecutive years

(1982-1989). We extracted 3 industries to be used as intra-industry samples (table 3) and we also pooled all the extracted firms into a single cross-industry sample. Only HK firms having accounts denominated in US or HK dollars were selected. Both intra-industry and cross-industry groups were examined. The number of firms per industry ranges from a minimum of 9 (Hotels, 1982) to a maximum of 69 (Properties, 1987). The number of firms in the cross-industry samples ranges over the years from 1982 to 1989, between 104 and 132.

Table 3 below shows the accounting aggregates tested. These are frequently employed as components of ratios. Where a sample contained sufficient negative values, two separate tests of lognormality were performed by taking the subset of positive values (denoted by an abbreviation, e.g., VAR) and then the absolute values of the negative subset (denoted by Ng VAR). This is because cross-sections of positive and negative values should be analysed separately as explained in the previous chapter.

2.2 Intra-Industry Results

In the examination of individual industries, 432 tests were carried out, corresponding to 18 different variables for each of the 3 selected industries, during a period of 8 years. Lognormality could not be rejected in most of these samples, as follows:

- two-parametric lognormality could not be rejected in 310 tests (72%);
- three-parametric lognormality could not be rejected in 44 tests (10%);
- the hypothesis of lognormality was rejected in 78 tests (18%).

Table 3 discriminates these results by variable.

The detailed results are displayed in tables 13 to 30 in the appendix.

Table 4 shows results by industry. These three industries (Properties, Consolidated Enterprises and Hotels) contained enough negative values to allow the testing of the absolute values of the negative subset.

The 78 tests rejecting lognormality (table 4) were closely observed one by one. Most of them belong to the Hotels industry which exhibit well detached clusters of firms or a concentration of cases near the median, leading to leptokurtosis.

2.3 Cross-Industry Results

The results of testing the pooled samples for lognormality are displayed in Tables 31 and 32 in the appendix. Lognormality was not rejected for 84% of the tests. Only one test showed traces of three-parametric lognormality.

Variable	Two parametric lognormal	Three parametric lognormal	Non lognormal
C	17	1	6
D	16	1	7
NI	21	2	1
OP	23	0	1
I	19	3	2
WC	23	0	1
EX	18	0	6
CL	19	0	5
S	19	0	5
TD	21	1	2
FA	11	12	1
DB	12	9	3
CA	21	0	3
NW	12	10	2
TA	19	5	0
Ng NI	12	0	12
Ng WC	19	0	5
Ng OP	8	0	16
Total	310	44	78

Table 3: For each variable, the table shows the number of tests leading to different results.

	Industry								
	Properties			Consolidated Enterp.			Hotels		
Var.	Two p logn	Three p logn	Non logn	Two p logn	Three p logn	Non logn	Two p logn	Three p logn	Non logn
C	8	0	0	7	1	0	2	0	6
D	8	0	0	6	1	1	2	0	6
NI	8	0	0	8	0	0	5	2	1
OP	8	0	0	8	0	0	7	0	1
I	8	0	0	8	0	0	3	3	2
WC	8	0	0	8	0	0	7	0	1
EX	8	0	0	8	0	0	2	0	6
CL	8	0	0	8	0	0	3	0	5
S	8	0	0	8	0	0	3	0	5
TD	8	0	0	7	1	0	6	0	2
FA	2	6	0	3	5	0	6	1	1
DB	7	1	0	4	4	0	1	4	3
CA	8	0	0	8	0	0	5	0	3
NW	6	2	0	0	8	0	6	0	2
TA	8	0	0	3	5	0	8	0	0
Ng OP	6	0	2	2	0	6	0	0	8
Ng NI	8	0	0	4	0	4	0	0	8
Ng WC	8	0	0	6	0	2	5	0	3
Total	133	9	2	106	25	13	71	10	63

Table 4: For each variable, the table shows the number of tests leading to different results by industry.

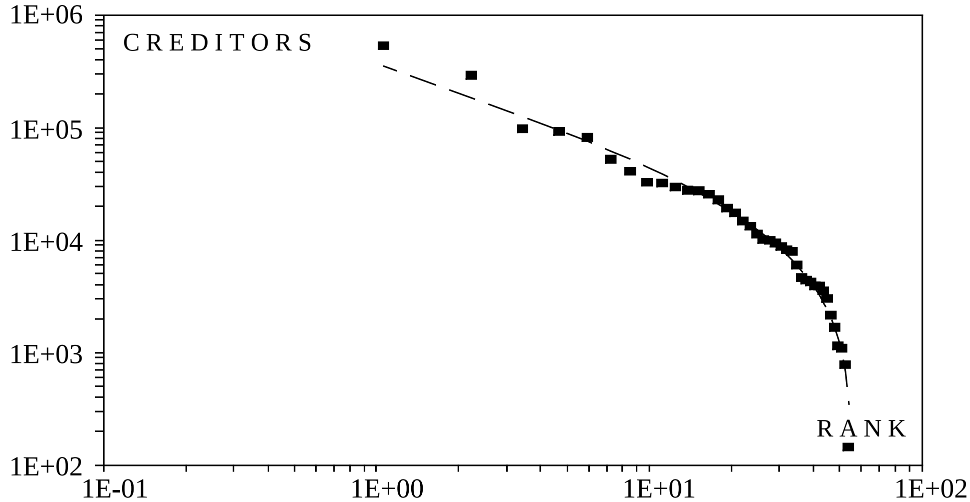

Figure 7: Relationship between log-rank (X-axis) and log-value (Y-axis). The dashed line is the lognormal deviate. Each point represents the Creditors item reported by a firm (1987).

An additional finding that is worth reporting, and that applies to the industry samples as well as the pooled cross-industry samples, concerns the positive kurtosis observed in all cases after transformation. The skewness and kurtosis of the aggregates were extreme, as expected. After a logarithmic transformation, the skewness vanished but all of the samples continued to exhibit traces of leptokurtosis.

2.4 Testing Other Transformations

This study also tested the possibility of obtaining normality when using transformations other than the logarithmic. The logarithmic transformation can be viewed as a way of removing positive skewness. It makes sense to ask whether the achieved reduction in skewness is appropriate. If less reduction is required, a square root or another root should be used instead. If more reduction is required, the Pareto distribution or another of its class should be used.

First, a scale of roots progressively approaching the effect of a logarithmic transformation was tested. We observed that there is progress towards normality for roots of increasing exponent and that symmetry is maximal when using logarithms. It may be noted that Ezzamel & Mar-Molinero (1990) reported an unpredictable distribution of ratios after applying similar transformations, which contrasts with the regularity observed in the underlying accounting variables.

The Pareto transformation, a more powerful transformation than the logarithmic in neutralizing positive skewness, was also tested. Pareto distributions occur if the relationship between observations and their rank in the sample is logarithmic. Where values are ranked from large to small, then log-values and log-ranks should be linearly related for the Pareto to hold.[3] However, a clear downward concavity of the distribution was observed in all tests. In general, firms occupying the middle of the rank were found to be about twice as large as that predicted by the Pareto distribution. Figure 7 shows an example of the relationship between logarithms of Creditors and logarithms of their rank. Observations follow much more closely the lognormal deviate (the dashed line) than a Pareto straight line. Ijiri & Simon (1977) reported the same concavity for US data.

2.5 Comparing Bounded and Unbounded Ratios

This section carries out the empirical comparison of bounded and unbounded ratios, stressing their different characteristics. Two sets of ratios are identified. In the first set, the denominator is a boundary to the numerator. The skewness of these ratios is smaller than expected in multiplicative data, suggesting that symmetrical or negatively skewed ratios reflect the existence of boundaries. Moreover, bounded ratios become skewed after being inverted, thus making it difficult to accept the hypothesis that the skewness of ratios stems from non-proportionality. In the second set of ratios, the denominator is not likely to bound the numerator. In this set, the estimates of skewness are in agreement with values expected for multiplicative data, showing that its origin is lognormal.

Besides classifying ratios as bounded or unbounded, the criterion adopted for selecting ratios was twofold: ratios in both sets should share as many items as possible and they should resemble those already tested by other authors. Eight years (1982-1989) were examined. Only positive values were included since, as mentioned above, cross-sections of positive and negative values may be seen as different populations.

2.5.1 Bounded Ratios Are Near Symmetry

Positive skewness should decrease in proportion to the strength of constraints affecting ratios. The nearer the numerator of these ratios is to the denominator, the farther should their distributions be from positive skewness. In order to test this hypothesis, 10 ratios were selected in which the numerator is bounded by the denominator. For each of them, $|\zeta|$ in formula (3) was calculated.

Tables 33 to 56 (appendix) display the selected ratios, their skewness and the value of $|\zeta|$. The values observed for skewness agree with those reported by other authors. As shown on the previous chapter, normalized distances below 2 denote significant constraints. Ratios such as CA/TA should be the most affected, as their $|\zeta|$ is small. In fact, these ratios exhibit negative skewness. Probably, this is because they are so strongly constrained that their distributions become skewed in the negative direction. Lognormal distributions are two-tailed. If the large right hand tail almost vanishes, the small left hand tail introduces negative skewness.

The values of $|\zeta|$ suggest that ratios like NW/TA or I/CA should be significantly affected, though less than those mentioned above. In fact, these ratios are almost symmetrical. This is probably because the large right hand tail of their distributions is shortened to an extent where it

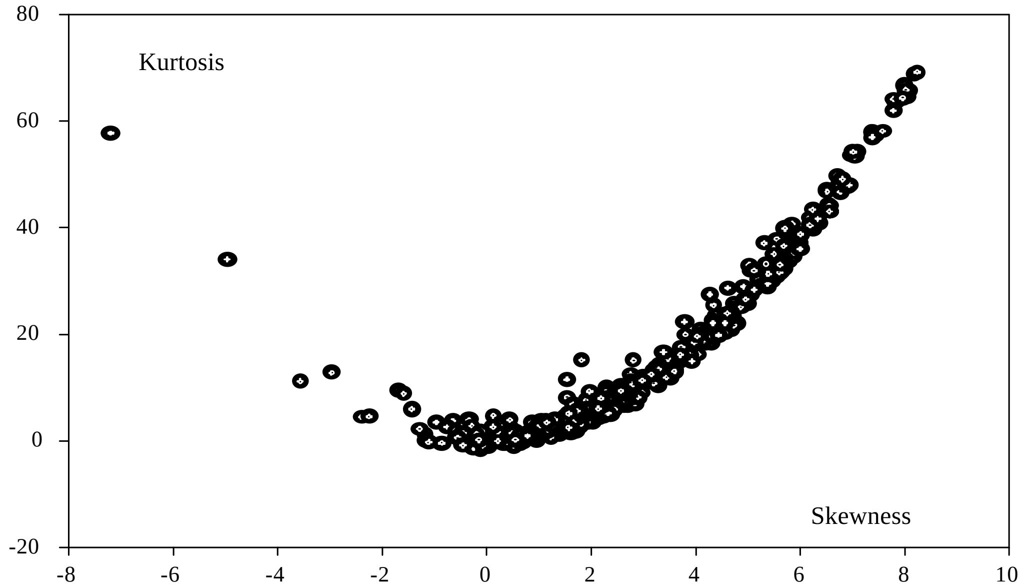

Figure 8: The functional relationship between skewness and kurtosis in unconstrained ratios. Each ratio is represented by a circled-cross sign.

is in balance with the left hand tail. Next, FA/TA or I/TA are affected to a smaller degree. Their skewness is positive but less than expected in multiplicative data. Finally, given $|\zeta|$, the constraint should be very small in ratios like C/TA and almost non-existent for OP/S. In fact, this reasoning is supported by the data as these ratios are very skewed.

2.5.2 Unbounded Ratios Are Broadly Lognormal

The 20 ratios for which there is no obvious constraint, are listed in tables 57 to 80 (appendix). Three facts emerge. First, these ratios are not far from lognormality. This can be ascertained by observing the strict relationship between their skewness and kurtosis, which is a typical feature of multiplicative data (Aitchison & Brown, 1957, pp. 8-9). Figure 8 is a graphical representation of tables 57 to 80 (appendix). It displays the regular curve that is formed by unconstrained ratios when skewness is plotted against kurtosis. Each ratio is represented by a circled-cross sign. This regularity is close to the relationship expected in lognormal variables.

Second, the inference that profitability ratios such as ROE are multiplicative would appear to contradict the findings of some other authors. The literature on the distribution of ratios seems to implicitly consider profits as additive, albeit non-normal (McLeay, 1986*a*; 1986*b*; Tippett, 1990; amongst others). Probably this is because, when studying profitability ratios, negative values are included in samples. However, according to our assumptions (stated in Chapter 1), where raw numbers take on positive and negative values across firms, then the distribution of negative values should be a negative mirror-image of the lognormal distribution. In that case, the overall distribution of items such as Earnings or Working Capital would be a juxtaposition of two lognormals. Ratios formed with these combined distributions might be markedly two-tailed. Long-tailed distributions such as Student's t or Cauchy (McLeay, 1986*b*) could fit them closely.

A third fact about unbounded ratios is that none of them is exactly lognormal, despite the strict lognormality of raw numbers. Lev & Sunder (1979, p. 204) and McLeay (1986*a*), when noticing that ratios of lognormal variables should also be lognormal, were referring to the theoretical case. Ratios are near lognormality but their logarithms are leptokurtic. The presence of leptokurtosis in log-ratios explains why, for some ratios, no transformation seems to succeed in approximating normality (Beaver, 1966; Ezzamel*et al.*, 1987; Ezzamel & Mar-Molinero, 1990).

2.6 Skewed Ratios and Non-Proportionality

The main reason for using ratios is to remove the influence of firm size from accounting variables thus making them comparable. In the course of their critiques of the practice of ratio analysis, Lev & Sunder (1979) and Whittington (1980) argued that size is only properly removed where the numerator and the denominator of the ratio are proportional. Accordingly, these authors advocated a regression rather than a ratio approach to remove the effect of size. Barnes (1982), added that non-proportionality probably also explained why the distribution of ratios is skewed. These views were shared by Lee (1985), Ezzamel *et al.* (1987), So (1987) and others. A continuing stream of research on the validity of ratios routinely implies that non-proportionality may have a role in explaining distributions of ratios.

However, since ratios are multiplicative and skewness is an expected quality of multiplicative data, then non-proportionality is not required for explaining skewness in ratios. In fact, if skewness were caused by non-proportionality, then ratios which are symmetrical should also be proportional. They should obviously remain proportional and symmetrical when inverted. The constraining mechanism predicts the contrary: reciprocals of symmetrical ratios should be very skewed.

Table 5 compares the skewness of CA/TA, NW/TA and FA/TA [4] (all industries together) with the skewness of their reciprocals. It can be seen that the reciprocals are distinctly multiplicative while the original ratios are not far from normality. Therefore, skewness in ratios is not originated by non-proportionality between the numerator and the denominator of ratios.

There is another reason why regressions should not be seen as a good alternative to ratios. Previous studies stressed the fact that regressions may add to financial measurement the possibility of modelling non-proportional relationships. However, regressions add more than just intercept terms: they also introduce, through the slope coefficient, an estimation of correlation between components. Such estimation, not present in traditional ratio analysis, substantially changes the nature and scope

	Ratio	Year 1982	1983	1984	1985	1986	1987	1988	1989
Not Inverted	CA/TA	0.79	0.48	0.57	0.63	0.50	0.49	0.89	0.88
	FA/TA	0.53	0.58	0.50	0.55	0.82	1.05	1.68	1.87
	NW/TA	-0.88	-0.56	-0.67	-0.69	- 0.53	-0.74	-0.70	-0.74
Inverted	TA/CA	10.62	10.29	8.30	8.51	9.95	10.08	6.75	9.32
	TA/FA	10.36	10.54	9.19	10.37	10.25	10.72	10.58	10.57
	TA/NW	5.31	5.96	3.76	4.31	4.45	9.47	8.15	6.06

Table 5: Skewness of three constrained ratios and their reciprocal. All industries.

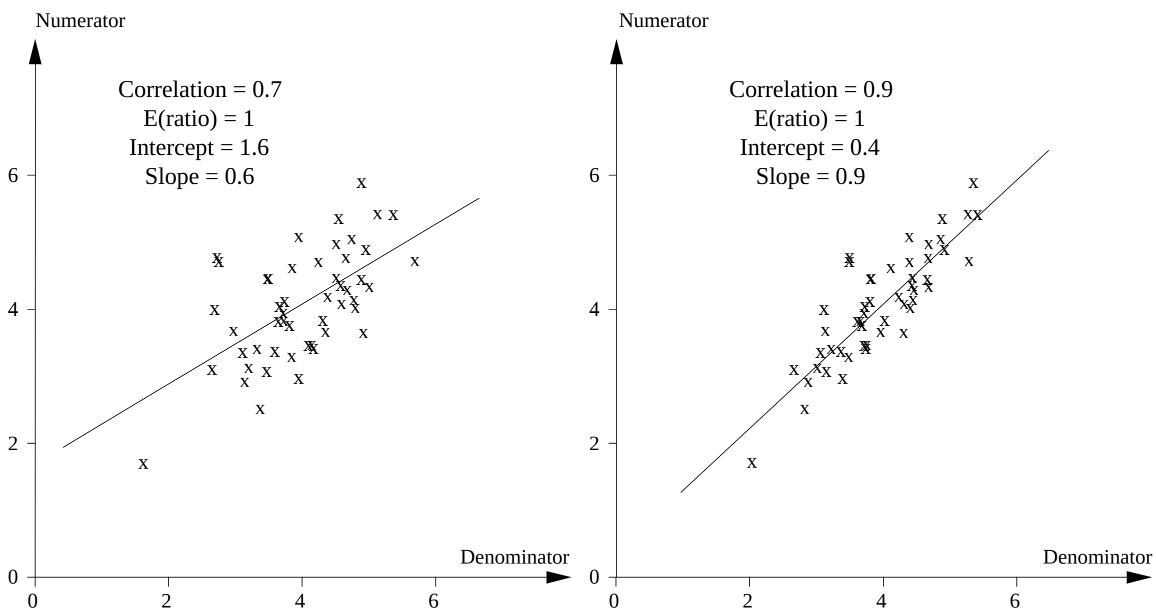

Figure 9: The two variables (called Numerator and Denominator) displayed on the left and on the right have the same expected values. Only the correlation between them is different and so, therefore, are the regression coefficients.

of financial measurement.

Since ratios are simple proportions, the estimation of ratio benchmarks such as industry norms only requires the estimation of the expected values of components. In contrast, in a regression, both the slope and the intercept term are influenced by the variance and co-variance of components (figure 9). The improvement offered by threshold ratios (4) and (5) over traditional analysis is the possibility of modelling expected proportions irrespective of correlation, thereby preserving the ratio's characteristics, which appear to be important to financial decision makers.

In the threshold ratio, for instance, non-proportionality is modelled as a constant displacement in the distribution of one of the components whereas the regression intercept term stems from the interaction between the two variables. Only when the relationship between components is deterministic (i. e., correlation is equal to 1) are the regression coefficients readily interpretable in the same way as the usual ratio measurement.[5]

Functionally, ratio (5) is also different from regression in that its 'intercept term', $-\delta_x P f_{y/x}$, is not constant. It depends not only on δ_x and P, but also on the residual $f_{y/x}$ in each case. The threshold in ratio (4), δ_y, might be viewed as similar to a regression intercept as it introduces a case-independent constant term. However, ratios (4) and (5) are essentially similar. On a logarithmic scale, they become, in the same order,

$$\log(y - \delta_y) - \log x = \mu + e_{y/x} \quad (6)$$

$$\log y - \log(x - \delta_x) = \mu + e_{y/x} \quad (7)$$

with $\mu = \log P$ and $e_{y/x} = \log f_{y/x}$. On this scale, corrections introduced by a similar δ in numerators or in denominators have the same magnitude. On the original scale, denominators correct for distortions limited by the range $\{0, 1\}$ whereas numerators correct for magnified distortions that are greater than 1. The basic difference remains that the estimation of threshold ratio parameters does not require consideration of the variance and co-variance of components.

There are two other differences between threshold ratios and regressions. First, like traditional ratios, threshold ratios may be treated as errors-in-variables models where appropriate. In contrast, regressions require the assumption that independent variables are explanatory, yet as Horrigan (1983) pointed out 'neither component [in ratios] is the exogenous, causal variable, nor is one component necessarily stochastic while the other is deterministic'. Second, threshold ratios are consistent with the multiplicative character of the components whereas the assumption underlying regressions is that variables are additive.

2.7 Concluding Comments

The findings of this study raise two important issues. First, the widespread lognormality of accounting aggregates suggests that the mechanism governing their cross-section distribution is general rather than particular to this or that item. In order to explain lognormality in accounting numbers, it might be sufficient to consider the growth of the firm as multiplicative with accounting variables reflecting, on average, a given proportion of firm size. Second, functional relationships between two lognormal variables may describe an expected proportionality of random effects, of which strict proportionality is just the simplest formulation. Therefore, besides ratios, other functional forms exist, capable of modelling the statistical characteristics of accounting numbers while removing the effect of size. For example, three-parametric lognormality suggests an obvious extension of ratios probably able to comprise non-proportionality as explained in the following chapter.

Our findings remove one major difficulty in understanding the statistical distribution of ratios: the existence of symmetry and negative skewness is explained as the effect of external boundaries such as accounting identities. The widespread lognormality of raw accounting numbers provides a privileged viewpoint from which ratios can be studied. First, it shows that ratios are expected to be multiplicative. Thus deviations from positive skewness, not deviations from symmetry, should be the main object of interest. Second, it unveils interesting features of ratios such as log-leptokurtosis. The findings of this study show that, after all, there is something regular and easy to understand in ratios.

2.8 Notes

1. See Ijiri & Simon (1977), for example.

2. For each test, δ was estimated by applying a modified version of the procedure suggested by Royston (1982, p. 123). The Shapiro-Wilk test produces a statistic, W, ranging from zero to one. Values of W approaching 1 mean increasing normality. Royston uses trial and error to find out which δ maximizes W. Using simulation, we noticed that the threshold should be estimated as the smallest δ able to attain a non-significant W, not as the δ yielding the largest W.

3. Where x is Pareto-distributed, then $\log x = \log M - \beta \times \log r$. r is the rank of x. The largest x is assigned the rank 1. M and β are parameters of the distribution.

4. The distribution of FA/TA is the mirror-image of CA/TA. This is because the two ratios add to 1. The same can be observed in the pair I/CA and QA/CA.

5. In this case, the slope is the proportion between average numerator and demominator and the intercept term is their difference.

Part II

Time-Series Theory and Evidence

Chapter 3

Discrete-Time Stochastic Ratios

Financial analysts have often worked closely with mathematicians. Indeed, mathematical models, especially of the type used to describe stochastic processes, are the backbone of financial economics. The following three chapters introduce the basic concepts required to study stochastic processes, applying them to an important topics in the field of Financial Analysis.

The present chapter provides some weak evidence on the stochastic characteristics of financial aggregates and the corresponding ratios extracted from accounting reports of firms in Hong Kong; in the following chapter, we introduce continuous-time stochastic processes and stochastic calculus. Finally, on the last chapter, we study the basic rule underlying the valid use of financial ratios, scale invariance, in the specific case of accounting aggregates which are continuous-time Markov processes.

The distinguishing characteristic of the second part of the thesis is the fact that each major topic is preceded by a summarisation of related, important concepts and developments in the field of stochastic processes. We hope that such summarisation may facilitate the reading and understanding of this study.

3.1 Stochastic Processes

A stochastic process is a random variable that changes in time. Formally speaking, a stochastic process is defined by a probability law for the evolution x_t of a variable x over time t. Furthermore, stochastic processes can be divided into two kinds: stationary and nonstationary.

In stationary processes the mean and the variance of x are continuous over time. An example of a stationary process is the height of an adult aged over 25. Although the expected height tomorrow depends partly on the height today, the expected height on January 1 of next year, as well as its variance, are largely independent of the height today and they should be equal to the expectation and variance of the height on January 1 two years from now, three years from now, etc.. In short, long term expected heights and variances converge to a constant. Another example is the mean-reverting process which will be discussed later.

In a nonstationary process the expectation and the variance (or just one of these) change with time. Stock prices are examples of nonstationary processes where the variance increases with time.

Stochastic processes which satisfy the *Markov property*, are called *Markov processes.* Markov processes are characterised by the fact that only the most recent event contains information useful to predict future events. These processes are used extensively as plausible approximations to the way variables such as stock prices or accounting incomes are generated since they embody the important economic intuition that managers and investors should be unable to estimate accurately future expected incomes thus being also unable to avoid exposure to risk. In the literature on financial ratios, Lev (1969) is an early example of their use. We further develop this important topic on section 3.3.1.

More formally, the Markov property requires that the probability distribution of the variable only depends on its current value, being independent from values on previous periods or from any other current information. Hence, the current value of the variable represents its best possible forecast.

One important instance where the Markov property holds is the time series of stock prices. It is possible to forecast the market price by using the current price since the current price reflects all the past information, as well as the market expectation regarding the underlying asset. If this were not the case, it would be possible to "beat" easily the market by cautiously studying the past price history.

What would be the best forecast of a stock price if the past information were taken into account? The Martingale model answers this question.

3.2 Martingales

The Martingale model captures the essence of a fair game, which is unbiased in all the aspects involved. In general, a Martingale is the stochastic process $\{P_t\}$ with the following condition, where t denotes time:

$$E[P_{t+1}|P_t, P_{t-1}, \ldots] = P_t \tag{8}$$

In other words, given the history of P_t, the expectation of the difference between P_{t+1} and P_t is zero. Therefore, according to Martingale model, the 'best' forecast of the stochastic process next period is simply the present one given the past history of the stochastic process. Returning to our stock price example, the Martingale model forecast for tomorrow's price is the same price for today, conditioned on the entire price history of the stock.

3.2.1 Mean-Reverting Processes

Suppose x_t is a stochastic process at discrete time t, the Mean-Reverting processes is defined:

$$x_t = \delta + \rho x_{t-1} + \zeta_t \tag{9}$$

where δ and ρ are constants, with $-1 < \rho < 1$ and ζ_t, the random term, is normally distributed with zero mean. This is a stationary process because the expected value of x_t, which is independent of the current value of x, is $\delta/(1-\rho)$ for any t.

This process is also known of the first-order auto-regressive process or AR(1). So far in this section, we have only discussed the discrete-time continuous-state stochastic process. (The process is continuous-state in the sense that ζ_t is normally distributed which plays the continuous role). There is also *discrete-time discrete-state process.* We should start off by the famous example of random walk.

3.2.2 Random Walk

A *discrete-time discrete-state random walk* is the process defined as follows: consider x_t, a random variable that begins at a known value x_0 and, at particular times $t = 1, 2, 3, \ldots$ etc., can take a jump of size 1 upwards or downwards, each with probability 1/2. Note that the jumps are mutually independent. Therefore, we can describe this random walk by the following equation:

$$x_t = x_{t-1} + \epsilon_t \tag{10}$$

where ϵ_t is a random variable with probability distribution

$$p(\epsilon_t = 1) = p(\epsilon_t = -1) = 1/2 \qquad (t = 1, 2, \ldots)$$

x_t is a discrete-state process because the jumps can only take place at specific time, for instance, $t = 1, 2, 3 \ldots$. Note that the possible range and variance for x_t increases with t. Therefore, x_t is a nonstationary process. Moreover, if the upward probability is changed to p and the downward probability to q, where $p > q$ and $p + q = 1$, then the expected value of x_t increases with t and is greater than zero. This is called a *random walk with drift.* Another approach is that the jump, ϵ_t, at each discrete time t can be normally distributed with zero mean and standard deviation of σ. As a result, the random walk models a discrete-time but nonstationary continuous-state stochastic process.

Both the random walk and the AR(1) processes mentioned above are Markov processes, as they satisfy the Markov property so that the probability distribution for x_{t+1} depends only on x_t but not on any other information. We shall need this assumption when we go further to the continuous model of the stochastic process.

3.3 Weak Evidence on the Stochastic Characteristics of Financial Aggregates

This section provides some limited evidence on the stochastic characteristics of accounting aggregates and their ratios of financial reports in Hong Kong. Specifically, we compare two extreme situations, testing the likelihood or these variables obeying the Markov property as opposed to the existence of predictability.

The widest time-span obtained from the PACAP database for Hong Kong is only thirteen years. We managed to isolate fifty firms that can be suitable for time-series analysis and this is clearly a very reduced set. Therefore we cannot claim that hypotheses have been tested but just that, as mentioned, the likelihood of one type of process maybe, in each case, greater than the other.

3.3.1 Economic Reasons for Expecting Martingale-Type Processes

The interpretation of growth and decline of firms (and their extreme counterparts, survival and failure) depends heavily upon the stochastic process considered. Growth in a martingale occurs as frequently as decline; and either, once experienced, is permanent, on average. Growth in a process whose expectation is constant over time is nonexistent.

The implications of income variability for the survival of a firm depends upon the process which generates income. A constant-expectation finite-variance process implies the relative insignificance of variability in income. Value changes are relatively small — the value of the firm changes in the order of the size of the deviation in income, given the (known) expectation of the process. If the expectation of income is known and stationary (and it surely would become known), then investors face very little risk. It is difficult to see why individual firms would ever fail; the possibility of ruin seem to be avoidable by borrowing when the expectation of income is constant. While borrowing is not without cost, it is then difficult to see the *ex-ante* importance of variability to the valuation of the firm, conditional upon a (known) long-run expectation.

In contrast, a process whose expectation is not constant or a deterministic function of time, implies the importance of variability. For a martingale (that is, ignoring trend), there is a finite probability, which is a function of the variability of the process, that the expectation of income at some future time will be negative or zero, and that the firm will on average fail. The expectation of all future incomes is changed with each observation. Hence, investors face greater risk than under the other type of income process. Assuming a Bayesian viewpoint, the value of the firm should change in the order of a known proportionality factor times the change in income, reflecting the changed expected profitability.

Given that the nature of the income process has importance for the interpretation of growth in the incomes of firms, what is the prior evidence and what does it imply about the income generating process?

3.3.2 Prior Evidence

The original work of Little (1962) and the later and larger study of Little and Rayner (1966) indicate that successive growth rates in the incomes of British companies are random. Both studies use percentage changes in incomes to measure growth rates and are concerned with the period-to-period stability of those growth rates. As the authors recognize, the use of such period-to-period percentage changes biases the results towards randomness.

Lintner and Glauber (1967) also investigate the relationship between growth rates in successive periods. However, the periods are longer, namely 5 and 10 years, and the sample consists of the 309 U.S. corporations on the Compustat industrial tapes with positive dividends in each of the years 1946 to 1965 inclusive. Using logarithmic data, growth is measured as the slope coefficient in the regressions of various income variables on time over the 5 and 10 year periods (we use the same transformation on our study). Although they find very small cross-sectional correlation between the growth rates of successive periods, Lintner and Glauber are not prepared to accept the hypothesis that the successive growth rates are independent:

> "any conclusion to the effect that nothing but a table of random numbers is relevant to growth in the real world itself would be premature and unwise."

Brealey (1967) follows Lintner and Glauber in studying the incomes of U.S. corporations. However, he investigates changes in incomes instead of growth rates in incomes. From cross-sectional correlations of changes in incomes for various lags, Runs tests and financial analysts' predictions based on past accounting data only, Brealey effectively concludes that incomes follow a martingale. In his later book Brealey reviews his own study and that of Lintner and Glauber and does not change his conclusions.

This evidence suggests that incomes conform to the specific kinds of nonrandomness which are implicitly assumed by using specific tests. In our examination of the aggregates and ratios of Hong Kong firms, we do not impose such restrictive assumptions since it is difficult to hypothesize a specific form of time series behavior. While the theory of efficient markets may yield specific hypotheses for the time series behavior of market prices of securities, there is no such theory for firms' aggregates. The theory of the firm is comparative static rather than stochastic, and the properties of accounting measurement rules are not well understood. Consequently, our investigation is essentially a descriptive exercise.

3.3.3 Data

Data are from PACAP file for the thirteen years (1979-1991) from which only twelve years (1980-1991) are used after lagging. Firms with less than thirteen years of data are excluded from the sample because the estimating procedures are sensitive to both few and missing observations. As a consequence, fewer than the approximately 50 firms on the PACAP file are investigated, the number differing according to the specific variable used tested.

The effect of our sample selection procedure probably is to over-estimate the importance of trends which are imposed upon expectations. Because the PACAP file contains only survived firms, because it contains only large firms which are presumably older survivors than the average, and because we do not accept firms without early data and are presumably left with even older survivors than the PACAP average, we are likely to have fewer average decreases in wealth in our sample than have occurred over the time period under study.

The importance of this upward bias in income relative to genuine expectations of trend (due, say, to reinvestment) cannot be determined within the selected sample of firms. Results in the extant literature indicate that the effects are minimal.

3.4 Tests and Results

As we suggested above, due to our limited number of observations for each firm, the results we obtain may be sensitive to violations of the assumptions of each test. Analytical results for most tests are for "large" samples. We attempt to avoid both issues by comparing results pertaining to a number of different firms and for two different tests, the Runs test and the Serial Correlation test, both aimed at assessing the likelihood of independence in a series.

These two tests are applied to two differently manipulated series.

Changes: Results for $Y_a = Y_{t-1}$ (that is, for runs in signs of changes).

Residuals of Log Time-Regressions: For each variable considered, a time-series regression was fitted to its logs

$$\log Y_t = A + bt + \epsilon_t$$

Then, the serial correlation (Durbin-Watson Test) and the randomness of signs of ϵ_t were observed.

The interest of this design stems from the possibility of comparing two opposite situations for the same set of firms-years.

3.4.1 Runs Test

A weak test of independence is afforded by comparing the actual and expected numbers of runs in a series. The test is "weak" because it tests for independence conditional upon known probabilities of increases and decreases in income, which must be estimated from the sample itself. Thus it tends to fit the data too closely to the distribution which is assumed to generate the observations.

Furthermore, it tests a specific form of independence: the randomness of the sequential arrangement in signs of deviations within a finite series.

Wald and Wolfowitz calculate the exact distribution of the number of runs, R, under the assumption of independence, and show that:

$$\mu_R = \frac{2N_1N_2}{N} + 1,$$

and

$$\sigma_R^2 = \frac{2N_1N_2(2N_1N_2 - N)}{N^2(N+1)},$$

where μ_R and σ_R^2 are the expectation and variance of R respectively, N_1 is the number of cases when $Y_t > Y_a$ (for any Y_a), N_2 is the number of cases when $Y_t < Y_a$, and $N = N_1 + N_2$. Further assuming that both N_1 and N_2 are large, the statistic

$$Z = \frac{R - \mu_R}{\sigma_R}$$

is normally distributed, with limiting distribution normal (0,1). The normal approximation hold closely for $N_1, N_2 > 10$ (p. 289; pp. 414-16), which requires at least five more observations than our sample possesses.

The aggregates and ratios tested in year t, Y_t, are:

1. Change in the Current ratio (CA/CL).
2. Change in Dividends.
3. Change in the Gross Margin ratio.
4. Change in Net Income.
5. Change in Operating Profits.
6. Change in Market Price.
7. Change in Sales.

We incorporated a variable whose behaviour is well known, Market Price, for facilitating comparisons. In principle, this series should behave as a Random Walk.

Table 88 in the appendix shows the results of applying this test to each series. Table 6 is a summary of these results. It shows the quartiles and the number of the significant cases for the Runs test for the changes in the variables considered. Only dividends show signs of a Martingale-type of behavior for a few firms. The Current Ratio, through which liquidity is measured, shows no traces of this kind of randomness and the other accounting aggregates considered exhibit only one firm where the Runs test is significant. However, it should be noticed that the variable used to act as control, Market Price, also shows the same degree of randomness. This may, in term, be

	Chg. Curr. Rat.	Chg. Divid.	Chg. Gross Mg.	Chg. Net Inc.	Chg. Oper. Prof.	Chg. Mkt. Price	Chg. Sales
25 percentile	0.36	0.27	0.36	0.36	0.36	0.36	0.36
Median	0.76	0.72	0.76	0.76	0.76	0.76	0.76
75 percentile	0.76	0.93	0.76	0.94	0.76	1.00	0.76
No. of sig. cases	0	2	1	1	1	0	1

Table 6: This table shows the quartiles and the number of the significant cases for the Runs test for the changes of Current Ratio, Dividends, Gross Margin, Net Income, Operating Profit, Market Prices and Sales.

Slope

	Log of Sales	Log of Total Assets	Log of Current Assets	Log of Current Lia.	Log of Operat. Expense
25 percentile	0.040	0.054	0.025	0.033	0.032
Median	0.068	0.087	0.070	0.079	0.068
75 percentile	0.095	0.120	0.114	0.123	0.103
No. of sig. cases	28	41	33	31	21

Table 7: This table shows the quartiles and the number of the significant cases of Slope for Log of Sales, Log of Total Assets, Log of Current Assets, Log of Current Liabilities, and Log of Operating Profit.

caused by the fact that the number of periods in our samples is not sufficient to take as granted the normality of Z (an asymptotic result) let alone any inference based on it.

The independence of residuals was tested using regressions where time explains the logs of the following aggregates:

1. Residuals of Current Assets.
2. Residuals of Current Liabilities.
3. Residuals of Operating Expenses.
4. Residuals of Sales.
5. Residuals of Total Assets.

For each of these variables, the parameters of the regression (the slope and the intercept), together with the Coefficient of Determination (R^2) and the Durbin-Watson test for independence, are summarised on tables 7 to 10.

Detailed descriptions of these parameters and statistics for each individual regression and aggregate can be found in the appendix, in tables 83 to 87. Table 89 (also in the appendix) shows, for each series, the P-values obtained when applying the Runs test to the residuals of the same

Constant

	Log of Sales	Log of Total Assets	Log of Current Assets	Log of Current Lia.	Log of Operat. Expense
25 percentile	-183.31	-234.40	-221.67	-239.99	-200.37
Median	-130.85	-165.74	-133.45	-152.09	-131.35
75 percentile	-73.29	-100.93	-45.64	-60.67	-57.14
No. of sig. cases	26	39	33	31	21

Table 8: This table shows the quartiles and the number of the significant cases of Constant for Log of Sales, Log of Total Assets, Log of Current Assets, Log of Current Liabilities, and Log of Operating Profit.

Adjusted R Square

	Log of Sales	Log of Total Assets	Log of Current Assets	Log of Current Lia.	Log of Operat. Expense
25 percentile	0.296	0.533	0.235	0.195	0.192
Median	0.584	0.755	0.575	0.433	0.534
75 percentile	0.782	0.825	0.769	0.746	0.774
No. of sig. cases	28	41	33	31	21

Table 9: This table shows the quartiles and the number of the significant cases of Adjusted R Square for Log of Sales, Log of Total Assets, Log of Current Assets, Log of Current Liabilities, and Log of Operating Profit.

Durbin Watson

	Log of Sales	Log of Total Assets	Log of Current Assets	Log of Current Lia.	Log of Operat. Expense
25 percentile	1.03	0.59	1.00	1.05	1.01
Median	1.46	0.86	1.30	1.37	1.55
75 percentile	1.84	1.22	1.58	1.68	2.03
Comment	Spread	Largest Corr	Strong Corr	Strong	Spread
Positive Auto Correlation					

Table 10: This table shows the quartiles of the test, Durbin Watson, for Log of Sales, Log of Total Assets, Log of Current Assets, Log of Current Liabilities, and Log of Operating Profit.

	Residual of Current Assets	Residual of Current Lia.	Residual of Operat. Expenses	Residual of Sales	Residual of Total Assets
25 percentile	0.13	0.15	0.15	0.18	0.04
Median	0.36	0.38	0.40	0.40	0.05
75 percentile	0.57	0.76	0.80	0.84	0.19
No. of sig. cases	8	5	3	4	20

Table 11: This table shows the quartiles and the number of the significant cases for the Runs test for residuals of Current Assets, Current Liabilities, Operating Profit, Sales and Total Assets.

regressions. Table 11 summarises table 89, showing the quartiles and the number of the significant cases observed. It is clear that slopes are, in general, significant, denoting the existence of trends in individual firms time-histories. Moreover, such trend accounts for a large proportion of their variability.

When the Runs test is applied to such detrended series, the number of significant cases clearly increases. However, the Durbin-Watson test still denounces the existence of same degree of residual auto-correlation, thus making it difficult to consider residuals as purely random.

3.4.2 Serial Correlation

The analytical serial covariance of *changes* in equally-lagged drawings from an independently-distributed process is zero. The expectation of the computed serial correlation coefficient of large samples from an independent process also is zero. With large samples, the computed coefficient is insensitive on non-normality. With small samples (as it is indeed the case), and assuming normality,

$$\mu_S = -\frac{1}{N-1},$$

and

$$\sigma_S^2 = \frac{T-2}{(T-1)^2},$$

where μ_S and σ_S^2 are the expectation and variance of the computed coefficient S, N is the number of changes in the series, and T is N less the size of the lag ($N-1$ for successive differences). The distribution of S is approximately normal and hence the statistic

$$Z = \frac{S - \mu_S}{\sigma_S}$$

is also approximately $N(0,1)$.

The aggregates and ratios tested for serial correlation are:

1. Sales.

2. Net income.

	Sales	Operating Profit	Net Income	Dividends	Market Price
25 percentile	0.43	0.17	0.25	0.15	0.36
Median	0.57	0.47	0.41	0.32	0.54
75 percentile	0.91	0.79	0.61	0.59	0.69
No. of sign. cases	17	22	13	14	15

Table 12: For each firm, this table shows the quartiles and the number of the significant cases for the serial correlation test for Sales, Operating Profit, Net Income, Dividends and Market Price.

3. Operating Profit.

4. Dividends.

5. Market Price.

Table 90 in the appendix shows the results of the serial correlation for each series. Table 12 summarises table 90 showing the quartiles and the number of significant cases. Many of the series exhibit serial correlation and this feature is shared by all the aggregates or ratios, inclusive of an control variable, Market price. This result is compatible with the results of the Durbin-Watson test (table 10).

3.5 Interpretation of Results

Clearly, any significance obtained in the Runs test in the case of differences, indicates that the aggregate or ratio is near the Markov condition. In the opposite end, where an aggregate or ratio exhibits clearly visible trend or mean-reversion, then the residuals of the time-regression in logs should exhibit both a high degree of significance in the Runs test and a low level of auto-correlation. When comparing (for the same set of firms, years and aggregates) the two opposite situations described above, some interesting facts emerge:

1. The Markov condition seems, in general, not to be verified, even in the case of Market Prices.

2. There is clear evidence of the existence of trends and auto-correlation.

3. The above mentioned characteristics may vary across cases but they do not differ much inside each cases for the different variables tested.

As a conclusion, deterministic (as opposed to Markov-type) randomness prevails: although a few time-histories are clearly more like submartingales, most of them are just trends with randomness superimposed. This, in spite of the economic reasons (presented above) which suggest that time-histories of income and other related items should obey the Markov condition.

Chapter 4

Introduction to Stochastic Calculus and the Itô Lemma

Amongst the two types of Markov processes, discrete- and continuous-time, the literature on financial economics has stressed the latter for reasons related to mathematical tractability. Continuous-time Markov processes (or *diffusions*), in spite of being non-differentiable in the classical sense, nevertheless allow, in most of the cases with practical interest, the finding and manipulating of stochastic equivalents to differential equations. This is so thanks to a simple formula, the *Itô lemma* of overwhelming importance and which will be the object of close attention below.

This chapter is theoretical and pedagogical in nature. Its aim is to provide an introduction to stochastic calculus and to study, while doing so, how the classical rules governing differential calculus should be re-formulated when such rules no longer apply–the realm of Itô processes. In the first place, the chapter shows how diffusions are obtained from discrete-time stochastic processes.

4.1 The Continuous Model of a Random Walk – The Wiener Process

As studied in the previous chapter, the discrete-time random walk with the upward probability p and the downward probability q, may take a jump sized $\triangle h$. Hence, the random variable $\triangle x$ can move either upward or downward by $\triangle h$ at each period $\triangle t$. Note that x_t is a Markov process whose future values depend only on the current stage. Also, the probability of x moving up or down in each period is independent of the previous periods, which in short can be called independent increments. For example, if $x_t = 3$, then $x_{t+\delta t}$ can be $3 + \triangle h$ with probability p and $3 - \triangle h$ with probability q. Once we know x_t, all the previous values become insignificant to us. Therefore, a random walk is definitely a Markov process with independent increments since the probability distribution for $x_{t+\delta t}$

is independent of the values of $x_{t-\delta t}, x_{t-2\delta t}, \ldots$.

Next, we calculate the expected value of the future values of x and get $E[\triangle x] = (p-q)\triangle h$. The second moment of $\triangle x$ is

$$E[(\triangle x)^2] = p(\triangle h)^2 + q(-\triangle h)^2 = (p+q)(\triangle h)^2 = (\triangle h)^2.$$

Then the variance of $\triangle x$ is

$$\begin{aligned} Var[\triangle x] &= E[(\triangle x)^2] - (E[\triangle x])^2 \\ &= (\triangle h)^2 - (p-q)^2(\triangle h)^2 \\ &= [1-(p-q)^2](\triangle h)^2 \\ &= [1-(p+q)^2 + 4pq](\triangle h)^2 \\ &= 4pq(\triangle h)^2 \end{aligned}$$

Since the steps are independent of each other, the probability distribution for x_t is obtained from the binomial distribution. If n is the number of finite steps in a time interval of length t such that $n = t/\triangle t$, then the cumulated change $(x_t - x_0)$ has the mean $n(p-q)\triangle h = t(p-q)\triangle h/\triangle t$ and the variance $n[1-(p-q)^2](\triangle h)^2 = 4pqt(\triangle h)^2/\triangle t$. The results are parallel to the ones in the usual binomial distribution, where in n independent trials, a success in any one trial counts as 1 with probability p, on the contrary, a failure counts as 0 with probability q. The mean of the number of successes in n independent trials is np and the variance is npq in the usual binomial distribution. But in the case we just studied, in which the success counts as $\triangle h$ and a failure as $-\triangle h$, the mean and variance are found to be different from the usual binomial distribution, however, the interpretation for both cases are analogous.

Furthermore, we maintain the same mean and variance for the cumulated change $(x_t - x_0)$, and they remain independent by choosing the particular p, q, $\triangle h$:

$$\triangle h = \sigma\sqrt{\triangle t} \tag{11}$$

and

$$p = \frac{1}{2}[1 + \frac{\alpha}{\sigma}\sqrt{\triangle t}] \qquad q = \frac{1}{2}[1 - \frac{\alpha}{\sigma}\sqrt{\triangle t}] \tag{12}$$

Using the equations 11 and 12, one can calculate $p-q$:

$$p - q = \frac{\alpha}{\sigma}\sqrt{\triangle t} = \frac{\alpha}{\sigma^2}\triangle h.$$

Finally, let $\triangle t$ approach zero, the number of steps n goes to infinity and the binomial distribution reaches the normal distribution. Putting the expression for $\triangle h$ and $p-q$ into the formulas for the mean and variance of $(x_t - x_0)$. The mean becomes

$$t(p-q)\frac{\triangle h}{\triangle t} = t\frac{\alpha}{\sigma^2}\triangle h\frac{\triangle h}{\triangle t} = \alpha t$$

and the variance is

$$4pqt\frac{(\triangle h)^2}{\triangle t} = t[1-(\frac{\alpha}{\sigma})^2\triangle t]\frac{\sigma^2\triangle t}{\triangle t} \to \sigma^2 t.$$

Thus, for the limit as $\delta t \to 0$, the random walk converges to a stochastic process with mean αt and variance $\sigma^2 t$. This is called the *Brownian motion with drift*, where α is the drift.

Brownian motion with drift is also known as *Wiener process*. In general, the Wiener process is a continuous-time stochastic process with the following properties:

- The Markov property—the probability distribution for all future values of the process only depends on the current value, and it is immuned from the values on previous periods or by any other current information. Hence, the current value can be used to make the best forecast.
- Independent increments—the probability distribution of the change over any nonoverlapping time intervals is independent.
- normally distributed changes—in any finite time interval, changes of the process follow the normal distribution and their variance increase with time.

The definition of Wiener process can be stated in a more formal way. If $z(t)$ is a Wiener process, then any change in z, $\triangle z$, and the corresponding time interval $\triangle t$, follow the relationship:

$$\triangle z = \epsilon_t\sqrt{\triangle t},$$

where ϵ_t is a normally distributed random variable with a mean of zero and a standard deviation of 1. Moreover, since the random variable ϵ_t is uncorrelated of each other on any nonoverlapping time intervals, $E[\epsilon_t\epsilon_s] = 0$ for $t \neq s$. Hence, we can ensure that the values of $\triangle z$ for any two different time intervals are independent. In other words, $z(t)$ is a Markov process with independent increments.

Note that if $\triangle t \to 0$, $\triangle z$ becomes the increment of a Wiener process, dz, such that

$$dz = \epsilon_t\sqrt{dt}. \tag{13}$$

Since ϵ_t has mean of zero and standard deviation of 1, it is not hard to find $E(dz) = 0$, and $Var[dz] = E[(dz)^2] - (E[dz])^2 = dt$. One might argue that the time derivative for a Wiener process does not exist, i.e. $\triangle z/\triangle t = \epsilon_t(\triangle t)^{1/2}$, because it approaches infinity as $\triangle t \to 0$. We will return to this discussion later.

Returning to our example of the Brownian motion with drift, we can rewrite it as:

$$dx = \alpha dt + \sigma dz \tag{14}$$

where α is the drift parameter and σ is the variance parameter. It is not surprise to see that the expected value $E(\triangle x) = \alpha\triangle t$ and the variance $Var(\triangle x) = \sigma^2\triangle t$, which coincide the result we found earlier.

(14) is also called the simple Brownian motion with drift, which belongs to the family of *Itô processes*. In other words, Itô processes are the generalization of the Brownian motion.

4.2 The Generalization of Brownian motion – Itô processes

In the simple Brownian motion with drift, (14), α and σ are fixed coefficient. If we change them to some known functions of the current state and time, then the process becomes the Itô process. An Itô process $x(t)$ is a continuous stochastic process, written as

$$dx = a(x,t)dt + b(x,t)dz \tag{15}$$

where dz is the increment of a Wiener process and $a(x,t)$ and $b(x,t)$ are predetermined functions of the current state and time.

In order to see the mean and the variance of the Itô process, we should look at the behaviors of the increments dz. Because dz is ruled by the Wiener process, $E[dz] = 0$, therefore $E[dx] = a(x,t)dt$. Also note that when dt is infinitesimally small, terms with order $(dt)^2$ and $(dt)^{3/2}$ approach zero. This can be seen as an application of the Itô formula which we will discuss in the next section. The same also happens to the case for the variance of dx, where $Var[dx] = E[dx^2] - (E[dx])^2$, involving terms in dt, in $(dt)^2$ and in $(dt)(dz)$ or the same as $(dt)^{3/2}$. Since both $(dt)^2$ and $(dt)^{3/2}$ converge to zero, then the variance of dx is

$$Var[dx] = b^2(x,t)dt \tag{16}$$

From (16), we conclude that $a(x,t)$ is the expected instantaneous *drift rate* of the Itô process, where $b^2(x,t)dt$ is the instantaneous *variance rate*.

4.2.1 Geometric Brownian Motion

Geometric Brownian motion with drift is developed from the Itô process by substituting $a(x,t) = \alpha x$ and $b(x,t)dt = \sigma x$, where α and σ are constants. In short, a geometric Brownian motion can be described by the equation:

$$dx = \alpha x dt + \sigma x dz \tag{17}$$

The expected value of the geometric Brownian motion $x(t)$ is

$$E[x(t)] = x_0 e^{\alpha t}, \tag{18}$$

where x_0 is the initial value of $x(t)$, and the variance is

$$Var[x(t)] = x_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1). \tag{19}$$

The result from (18) is useful because the expected present discounted value of $x(t)$ over some period of time can be obtained from it, i.e.

$$\begin{aligned} E\left[\int_0^\infty x(t)e^{-rt}dt\right] &= \int_0^\infty x_0 e^{-(r-\alpha)t}dt \\ &= \lim_{k\to\infty}\int_0^k x_0 e^{-(r-\alpha)t}dt \\ &= \lim_{k\to\infty} x_0 \frac{e^{-(r-\alpha)t}}{-(r-\alpha)}\Bigg|_0^k \\ &= \lim_{k\to\infty} x_0 \frac{e^{-(r-\alpha)k}-1}{-(r-\alpha)} \\ &= \frac{x_0}{r-\alpha}. \end{aligned} \tag{20}$$

This is the formula which allows the calculation of perpetual rents or "perpetuities" where α has the meaning of a growth rates and $r$is the discount rate, the growth rate α is less than the discount rate r. We are going to see more examples for the application of (18) later in this chapter.

In general, the value of x in the geometric Brownian motion has no restriction. However, in reality, there are always some stochastic processes $x(t)$ needed to be modified in order to satisfy some particular requirements. For instance, if a stochastic process models price, it would be impossible to consider a negative price. To ensure that the price positive, we take the logarithm of the original stochastic process. Similarly, supposing that $x(t)$ follows (17), the transformation of $x(t)$, $F(x) = \log x$ is the simple Brownian motion with drift:

$$dF = (\alpha - \frac{1}{2}\sigma^2)\,dt + \sigma\,dz. \tag{21}$$

we save the derivation of (21) for later in the chapter. It is easy to see that $\log x_{t+\delta t} - \log x_t$ is normally distributed with mean $(\alpha - \frac{1}{2}\sigma^2)t$ and variance $\sigma^2 t$.

In the previous section, we have seen that the limit of a random walk is the Wiener process. Indeed, in the beginning of the chapter, we have also learned the other type of discrete-time stochastic process, namely AR(1). What would be the continuous model of the AR(1)? To answer this, we should turn to next section.

4.2.2 The Continuous Model of the First-Order Autoregressive Process – Ornstein-Uhlenbeck Process

As we mentioned before AR(1), a subset of *Mean-Reverting process*, has a different nature from random walk and Wiener process, and therefore, is a stationary stochastic process. This is confirmed by the property of mean-reverting process, which says no matter what the current state of the random variable is, in the long run, it tends to revert to a certain level.

Let us recall (9),

$$x_t = \delta + \rho x_{t-1} + \zeta_t.$$

By choosing carefully of δ and ρ and introducing a constant η, the usual level of x, $\overline{x}$, and a less general type of normally distributed random variable ϵ_t, instead of ζ_t, we can rewrite (9) as

$$x_t - x_{t-1} = \overline{x}(1 - e^{-\eta}) + (e^{-\eta} - 1)x_{t-1} + \epsilon_t, \tag{22}$$

where ϵ_t has mean zero but a standard deviation of σ_ϵ, so that the variance of ϵ_t, σ_ϵ^2, is

$$\sigma_\epsilon^2 = \frac{\sigma^2}{2\eta}(1 - e^{-2\eta}).$$

We can use data obtained from discrete time to estimate the values of the parameters in (22) by running the regression:

$$x_t - x_{t-1} = a + bx_{t-1} + \epsilon_t.$$

The regression results in $\overline{x} = -\hat{a}/\hat{b}$, $\eta = -\log(1 + \hat{b})$. Besides, we can also calculate the estimated variance $\hat{\sigma}^2$, for $x_t - x_{t-1}$:

$$\hat{\sigma}^2 = \hat{\sigma}_\epsilon^2 \sqrt{\frac{\log(1 + \hat{b})}{(1 + \hat{b})^2 - 1}},$$

where $\hat{\sigma}_\epsilon$ is the standard error of the regression.

Now we will see what happens to the first-order autoregressive process. We can show that the process given by (22) satisfies the following stochastic differential equation:

$$dx = \eta(\overline{x} - x)dt + \sigma dz. \tag{23}$$

where $\overline{x}$ is the "normal" level of x, a level that x tends to draw back to. The constant η can be interpreted as the "speed" of reversion. (23) is the mean-reverting process in continuous time, which is also called *Ornstein-Uhlenbeck process.*

Inspecting the expected value of x at any future time t from (23),

$$E[x_t] = \overline{x} + (x_0 - \overline{x})e^{-\eta t}, \tag{24}$$

where x_0 is the current value of x, and the variance of $(x_t - \overline{x})$,

$$Var[x_t - \overline{x}] = \frac{\sigma^2}{2\eta}(1 - e^{-2\eta t}), \tag{25}$$

we see that when $t \to \infty$, $E[x_t] \to \overline{x}$ and the variance becomes $\frac{\sigma^2}{2\eta}$. In addition to that, the greater value of η, future value of x (x_t) is closer to the "normal" level of x ($\overline{x}$). Eventually, when $\eta \to \infty$, $Var[x_t] \to 0$, and therefore, x_t is kept at the level of $\overline{x}$, i.e. $x_t = \overline{x}$. On the contrary, the smaller η it is, the larger fluctuation for x around $\overline{x}$. If $\eta \to 0$, $Var[x_t] \to \sigma^2 t$, which goes without bound, then the process becomes a simple Brownian motion. Note that since the expected change in x ($E[x_t - \overline{x}]$) depends on the difference between x and $\overline{x}$, the Ornstein-Uhlenbeck process, even following the Markov property, does not have independent increments. Consequently, in the next time interval, x is more likely to fall if $x > \overline{x}$ and vice versa.

4.3 Itô's Lemma

At the end of the section 4, we have noted that the time derivative does not exist. In general, any Itô process is not differentiable. In this case, *Itô's Lemma* is here to solve our problem.

To begin Itô's Lemma, we should first state the governing multiplication rules:

$$(dt)^2 = 0 \tag{26}$$

$$(dt)(dz) = 0 \tag{27}$$

$$(dz)^2 = 1\,dt. \tag{28}$$

Taylor series expansion will be used to develop Itô Lemma. Suppose that $x(t)$ follows (15), and consider a function that is at least twice differentiable in x and once in t. The total differential of the function, dF is:

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2 + \frac{1}{6}\frac{\partial^3 F}{\partial x^3}(dx)^3 + \ldots. \tag{29}$$

The terms with the order greater than 2 will all approach zero at the limit $t \to 0$. To see this, we first calculate $(dx)^2$ using (15):

$$(dx)^2 = a^2(x,t)(dt)^2 + 2a(x,t)b(x,t)(dt)^{3/2} + b^2(x,t)dt. \tag{30}$$

After applied the multiplication rules (26), 27) and (28), the rate of terms in $(dt)^{3/2}$, or the same as $(dt)(dz)$, and $(dt)^2$ are all equal to zero, so $(dx)^2$ becomes:

$$(dx)^2 = b^2(x,t)dt.$$

For the term in $(dx)^3$ from (29), we can make the same expansion and get an expression all involving power of dt greater than 1. By applying the multiplication rules (26), 27) and (28) once more, we get $(dx)^3$ is equal to zero. Thus, $(dx)^3$ can be ignored and also the rest of the higher order term in (29). Hence, Itô's Lemma for one Itô process is that:

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial x}dx + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2. \tag{31}$$

Similarly, by substituting (29) for dx, dF can also be expressed as:

$$dF = [\frac{\partial F}{\partial t} + a(x,t)\frac{\partial F}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 F}{\partial x^2}]dt + b(x,t)\frac{\partial F}{\partial x}dz. \tag{32}$$

Itô's Lemma is also applicable to functions of several Itô processes but we will need additional multiplication rule for it, such that:

$$(dz_i)(dz_j) = \rho_{ij}\,dt, \tag{33}$$

where ρ_{ij} is the instantaneous correlation coefficient between dz_i and dz_j. If there are m Itô processes and $F = F(x_1, \ldots, x_m, t)$ is the function for these Itô processes and time t, where

$$dx_i = a_i(x_1, \ldots, x_m, t)dt + b_i(x_1, \ldots, x_m, t)dz_i, \qquad i = 1, \ldots, m \tag{34}$$

then Itô lemma becomes:

$$dF = \frac{\partial F}{\partial t}dt + \sum_i \frac{\partial F}{\partial x_i}dx_i + \frac{1}{2}\sum_i\sum_j \frac{\partial^2 F}{\partial x_i \partial x_j}dx_i dx_j. \tag{35}$$

Same as the previous case, dF can be expanded by putting (34) and (33) into the above formula as

$$\begin{aligned} dF &= [\tfrac{\partial F}{\partial t} + \textstyle\sum_i a_i(x_1,\ldots,x_m,t)\tfrac{\partial F}{\partial x_i} + \tfrac{1}{2}\sum_i b_i^2(x_1,\ldots,x_m,t)\tfrac{\partial^2 F}{\partial x_i^2} \\ & \textstyle\sum_{i\neq j}\rho_{ij} b_i(x_1,\ldots,x_m,t) b_j(x_1,\ldots,x_m,t)\tfrac{\partial^2 F}{\partial x_i \partial_j}]dt \\ & + \textstyle\sum_i b_i(x_1,\ldots,x_m,t)\tfrac{\partial F}{\partial x_i}dz_i. \end{aligned} \tag{36}$$

It will be easier for us to comprehend the dynamics of Itô's Lemma by going through a few examples.

Example 1: **Geometric Brownian Motion.** In section 4.2.1, we saw a geometric Brownian motion $x(t)$ can be transformed as $F(x) = \log x$ and the increment of the process, dF, is given by (21). Now, we will use Itô's Lemma to fill the gap in deriving dF.

Observe that $\frac{\partial F}{\partial t} = 0$, $\frac{\partial F}{\partial x} = \frac{1}{x}$, and $\frac{\partial^2 F}{\partial x^2} = -\frac{1}{x^2}$. Putting the corresponding terms in (31), we have

$$\begin{aligned} dF &= \tfrac{1}{x}dx - \tfrac{1}{2x^2}(dx)^2 \\ &= \tfrac{1}{x}[\alpha\, dt + \sigma\, dz] - \tfrac{1}{2x^2}(\sigma x\, dz)^2 \\ &= \alpha\, dt + \sigma\, dz - \tfrac{1}{2}\sigma^2\, dt \\ &= (\alpha - \tfrac{1}{2}\sigma^2)\, dt + \sigma\, dz \end{aligned} \tag{37}$$

The result is consistent with (21). We conclude that over any finite time interval T, $\log x_{t+\delta t} - \log x_t$ is normally distributed with mean $(\alpha - \frac{1}{2}\sigma^2)\,T$ and variance $\sigma^2\,T$.

Example 2: **Correlated Brownian motion.** This time, there are two geometric Brownian motions $x(t)$ and $y(t)$, so that

$$dx = \alpha_x\, x\, dt + \sigma_x\, x\, dz_x$$

$$dy = \alpha_y\, y\, dt + \sigma_y\, y\, dz_y$$

with $\mathcal{E}[dz_x dz_y] = \rho$. $x(t)$ and $y(t)$ follow the function $F(x,y) = xy$, and then once again $G = \log F$.

Discover that $\frac{\partial^2 F}{\partial x^2} = \frac{\partial^2 F}{\partial x^2} = 0$ and $\frac{\partial^2 F}{\partial x \partial y} = 1$. Substituting them into (35), we have

$$\begin{aligned} dF &= x\, dy + y\, dx + dx\, dy \\ &= x(\alpha_y\, y\, dt + \sigma_y\, y\, dz_y) + y(\alpha_x\, x\, dt + \sigma_x\, x\, dz_x) \\ &\quad +(\alpha_x\, x\, dt + \sigma_x\, x\, dz_x)(\alpha_y\, y\, dt + \sigma_y\, y\, dz_y) \\ &= (\alpha_y\, xy\, dt + x\, \sigma_y\, xy\, dz_y) + (\alpha_x\, xy\, dt + y\, \sigma_x\, xy\, dz_x) \\ &\quad +(\alpha_x\, \alpha_y xy\, (dt)^2 + \sigma_x\, \alpha_y\, xy\, dt\, dz_x \\ &\quad +\alpha_x\, \sigma_y\, xy\, dt\, dz_y + \sigma_x\, \sigma_y\, xy\, dz_x\, dz_y). \end{aligned} \tag{38}$$

Again, by applying the multiplication rules (26), 27) (28) and (33) to (38), we get:

$$dF = (\alpha_x + \alpha_y + \rho\, \sigma_x\, \sigma_y)F\, dt + (\sigma_x\, dz_x + \sigma_y\, dz_y)F, \tag{39}$$

where ρ is the instantaneous correlation coefficient between dz_x and dz_y. It is obvious that $F(x, y)$ in (39) is a geometric Brownian motion. Finally, consider $G = \log F$. This is similar to example 1, so we get

$$dG = (\alpha_x + \alpha_y - \frac{1}{2}\sigma_x^2 - \frac{1}{2}\sigma_y^2)dt + \sigma_x \, dz_x + \sigma_y \, dz_y. \tag{40}$$

(40) is also analogous to example 1 that over any time interval T, the change in $\log F$ follows the normal distribution with mean $(\alpha_x + \alpha_y - \frac{1}{2}\sigma_x^2 - \frac{1}{2}\sigma_y^2)\, T$ and variance $(\sigma_x^2 \, dz_x + \sigma_y^2 \, dz_y + 2\rho\sigma_x \, \sigma_y)\, T$

Example 3: **Present Discounted Value.** We have learned how to calculate the expected present discounted value for a geometric Brownian motion in section 4.2.1. Now we will repeat it again for $F(x) = x^\theta$, where $x(t)$ certainly is a geometric Brownian motion. According to Itô's Lemma

$$\begin{aligned} dF &= \tfrac{\partial F}{\partial t} dt + \tfrac{\partial F}{\partial x} dx + \tfrac{1}{2} \tfrac{\partial^2 F}{\partial x^2} (dx)^2 \\ &= \theta \, x^{\theta - 1} [\alpha \, x \, dt + \sigma \, x \, dz] + [\tfrac{1}{2}\theta \, (\theta - 1) \, x^{\theta - 2}] \, (\sigma \, x \, dz)^2 \\ &= (\theta \, \alpha \, x^\theta \, dt + \theta \, \sigma \, x^\theta \, dz) + [\tfrac{1}{2}\theta \, (\theta - 1)] \, \sigma^2 \, x^\theta \, dt \\ &= [\theta \, \alpha + \tfrac{1}{2}\theta \, (\theta - 1) \, \sigma^2] \, F \, dt + \theta \, \sigma \, F \, dz. \end{aligned} \tag{41}$$

(41) once again is a geometric Brownian motion. Hence, we can apply the result from (20) in section 4.2.1 to attain the expected present discounted value

$$\begin{aligned} E\left[\int_0^\infty F(x(t)) e^{-rt} \, dt \right] &= \int_0^\infty F(x_0) \, \exp[-(r - \theta \, \alpha - \tfrac{1}{2}\theta \, (\theta - 1) \, \sigma^2) \, t] \, dt \\ &= \tfrac{x_0^\theta}{r - \theta \, \alpha - \frac{1}{2}\theta \, (\theta - 1) \, \sigma^2}, \end{aligned} \tag{42}$$

providing $r > \theta \, \alpha + \frac{1}{2}\theta \, (\theta - 1) \, \sigma^2$. After the above examples we are indeed prepared to study the specific case of ratios.

Chapter 5

Application of Stochastic Calculus to the Validity of Financial Ratios

We mentioned before that, for reasons related to mathematical tractability, the literature on financial economics has paid great attention to continuous-time Markov processes, using the Itô lemma extensively as a tool to obtain useful closed-form solutions. The field of Financial Analysis is not an exception to this trend. In recent years, a considerable volume of research has emerged on the continuous time properties of financial ratios. The seminal work and procedures were laid down by Lev (1969).

This chapter explores the contribution that continuous time stochastic calculus models can make to this area. We begin by assuming that the aggregates from which the ratio is constructed are generated by a geometric Brownian motion. This assumption implies that the ratio itself will be both lognormally distributed and a non-linear function of time. We then turn to Lev's (1969) partial adjustment model. Using the properties of ratios generated by these two standard processes, we then examine the reasonableness or otherwise of the proportionality assumption (Lev & Sunder, 1979; Whittington, 1980), offering alternative definitions of scale-invariance, applicable to diffusions.

5.1 Geometric Brownian Ratios

Given two financial aggregates, neither of which can take on negative values, y and x, we suppose that both y and x evolve as geometric Brownian motions. Hence, for y, we have:

$$\frac{dy}{y} = \mu dt + dZ(t) \tag{43}$$

where μdt is the instantaneous mean rate of growth in y and $dZ(t)$ are white noise processes with variance σ^2. A process is said to constitute white noise if, over the successive time intervals $[t,\, t+dt]$, it is an independently and identically distributed normal variate with mean zero and instantaneous variance given by $\sigma^2 dt$. Occasionally, $dZ(t)$ is given the alternative representation $dZ(t) = \sigma z\sqrt{dt}$, where z is distributed as a standard normal variate and, as such, has mean zero and unit variance. Similarly, for x, we have:

$$\frac{dx}{x} = \theta dt + dQ(t) \tag{44}$$

where θdt is the instantaneous mean rate of growth in x and $dQ(t)$ is white noise with variance δ^2. Again $dQ(t)$ is occasionally given the alternative representation $dQ(t) = \delta q\sqrt{dt}$, where q is distributed as a standard normal variate. Applying Itô's lemma to the ratio

$$r = \frac{y}{x} \tag{45}$$

implies that the evolution of r is governed by the stochastic differential equation

$$\frac{dr}{r} = [\mu - \theta\rho\sigma\delta + \delta^2]dt + dZ(t) - dQ(t) \tag{46}$$

where $[\mu - \theta\rho\sigma\delta + \delta^2]dt$ is the instantaneous mean rate of growth in r whilst $dZ(t)$ and $dQ(t)$ are white noise. Note that the instantaneous variance of the rate of growth in r is:

$$\begin{aligned} \text{Var}[dZ(t) - dQ(t)] &= \text{Var}[dZ(t)] + \text{Var}[dQ(t)] - 2\text{Cov}[dZ(t), dQ(t)] \\ &= [\sigma^2 + \delta^2 - 2\rho\sigma\delta]dt \end{aligned} \tag{47}$$

where Var[.] is the variance of the relevant random variable. Cov[...] is the covariance between the two random variables and ρ is the Pearson product moment correlation coefficient between $dZ(t)$ and $dQ(t)$. Using standard techniques, it can be shown that the solution to the above stochastic differential equation is:

$$r(t) = r(0)\exp[(\mu - \frac{1}{2}\sigma^2)t - (\theta - \frac{1}{2}\delta^2)t + Z(t) - Q(t)] \tag{48}$$

where Z(t) is called a Wiener process with zero mean and variance $\sigma^2 t$. Similarly, $Q(t)$ is a Wiener process with zero mean and variance $\delta^2 t$. This implies that $r(t)$ is lognormally distributed with mean (Aitchison and Brown, 1957, p. 8):

$$E[r(t)] = r(0)\exp[(\mu - \theta + \delta^2 - \rho\sigma\delta)t] \tag{49}$$

Since the expected value for the ratio is a function of time, proportionality cannot be verified. It is also interesting to note that, even when $\mu = \theta$, we can still expect systematic drift in the ratio.

5.2 Mean-Reverting Ratios

In a seminal paper, Lev (1969) argued that financial ratios follow a partial adjustment model. The distinguishing characteristic of this model is explained by Lev (1969, p.292) in the following terms:

> . . . when the firm observes a deviation between its ratio and the industry mean.... it will adjust its ratio in the next period.... so that the observed deviation will be partially eliminated.

To illustrate the type of stochastic process which is consistent with this model, suppose our two previously considered financial aggregates evolve in accordance with the following stochastic differential equations:

$$\frac{dy}{y} = -\lambda \log \frac{y}{g(t)} dt + dZ(t) \tag{50}$$

and

$$\frac{dx}{x} = -\lambda \log \frac{x}{h(t)} dt + dQ(t) \tag{51}$$

As previously, $dZ(t)$ and $dQ(t)$ are white noise processes with variance σ^2 and δ^2 respectively. These processes are elastic random walks in the sense that both y and x have a greater tendency to revert to their normal values of $g(t)$ and $h(t)$, respectively, the further they are removed from them. In this respect $\lambda > 0$ is a parameter which measures the intensity with which the financial aggregates are drawn back to their normal values. Hence should $y > g(t)$, then $\log[y/g(t)]$ will be positive and the instantaneous expected rate of growth in y will be negative. Similarly, when $y < g(t)$, the instantaneous expected rate of growth in y will be positive. Using (45) and Itô's lemma, it follows that the evolution of the ratio of the two financial aggregates is governed by the following stochastic differential equation:

$$\frac{dr}{r} = [(\delta^2 - \rho\sigma\delta) - \lambda \log \frac{r}{\mu(t)}]dt + dZ(t) + dQ(t) \tag{52}$$

where $\mu(t) = g(t)/h(t)$ is the 'normal' value to which the ratio reverts and ρ is the Pearson product moment correlation coefficient between $dZ(t)$ and $dQ(t)$. Analysis similar to that employed in the previous section shows that the instantaneous mean rate of growth in the ratio amounts to:

$$[(\delta^2 - \rho\sigma\delta) - \lambda \log(r/\mu)]dt$$

Whilst the instantaneous variance is:

$$[\sigma^2 + \delta^2 - 2\rho\sigma\delta]dt$$

Using standard techniques, it may be shown that the solution to this stochastic differential equation is:

$$r(t) = \mu \exp[\gamma(t) - \frac{\sigma^2 - \delta^2}{2\lambda}] \tag{53}$$

where:

$$\gamma(t) = \mathrm{e}^{-\lambda t} \int_0^t \mathrm{e}^{\lambda s} dV(s) \tag{54}$$

is an additive elastic random walk with:

$$dV(s) = dZ(s) - dQ(s).$$

(53) implies that $r(t)$ is lognormally distributed (Aitchison and Brown, 1957, p.8). Further, it may also be shown that the proportionality assumption implicit in much of ratio analysis is also violated in this instance since co-variance introduced by the extra term present in the Itô formula means that same degree of non-proportionality will exist.

5.3 Stochastic Scale Invariance

Previous sections seem to show that financial ratios cannot be validly used in practice. In fact, wherever ratios are used, the implicit assumption is that the relationship between the numerator and the denominator is expected to remain constant no matter the changes observed in the ratio components, i.e.

$$\frac{Y}{X} = \text{Constant.}$$

For instance, the popular benchmark whereby the amount of Current Assets is expected to be twice as large as Current Liabilities is deemed to hold whether the figures involved are as large as \$ 2bn:\$ 1bn or as small as \$ 200:\$ 100. It is this which prompted authors such as Lev & Sunder (1979) and Whittington (1980) to state that the most basic requirement of ratio validity is proportionality between Y and X. Furthermore, these same authors also examined two conditions implicit in proportionality, namely a linear relationship between Y and X and the presence of a zero intercept. Any form of systematic non-proportionality such as the one outlined in the previous section will render ratios use invalid.

In order to conclude as to whether it is possible for ratios to be validly used, the first step consists of re-writing the above formula in a more general way. The above conditions for the validity of ratios emphasise the relationship between Y and X and not the way changes in Y should relate to changes in X so that the ratio remains constant. Yet it is obvious that, when Y is proportional to X, the rate of change of Y with respect to X remains constant and similar to the ratio itself. For ratios to be valid, therefore, the following relationship must hold:

$$\frac{Y}{X} = \frac{dY}{dX}$$

where dY, dX are any related changes or differences observed in Y and X. This formulation is the differential equivalent to Y/X = Constant. It fully encompasses the traditional definition of ratio validity, having the important advantage of explicitly showing a requirement of proportionality hitherto unnoticed. In fact, by re-arranging terms, the above identity becomes:

$$\frac{dY}{Y} = \frac{dX}{X} \tag{55}$$

Equality (55) shows that, implicit on proportionality, there is a condition, *scale invariance*, whereby the rate of change in Y should be equal to the rate of change in X. In other words, (55) says that ratios are valid only if the proportionate changes in the numerator and denominator are expected to be identical. For instance, when comparing firms in cross-section, if the Current Assets figure is expected to be three times larger in one firm than in another then this should also be the case for Current Liabilities or any other variable which is potentially useful as a component of a ratio. Similarly, in a time-series analysis, (55) implies that any variable eligible as a ratio component is expected to grow at the same rate. If, say, Sales is expected to grow by 12% in a given year, then Earnings should also be expected to grow by 12% during the year. It should not be surprising that the validity of ratios is conditional on the equality of proportionate changes in variables. Since ratios are scales, they are valid only where the scaling of the data makes sense and this implies scale invariance as a property of such data.

Scale invariance, as defined by equality (55) applies to deterministic changes but, as demonstrated in the previous section, in general it is not verified for stochastic changes in continuous-time Markov processes. However, it should not be concluded that it is impossible to observe scale invariance (and thence validly used ratios) in the expected relationship between stochastic variables.

The reason why (55) fails to encompass such variables is that it equates two effective rates of change (i.e. rates of change which are inherently discrete), whereas an equality between two continuous rates is now required. A formulation of the scale invariance condition which is robust regarding the nature of the variable (i.e. deterministic or stochastic, continuous or discrete) is obtained by equating expected continuous rates of change, as follows:

$$E\left[\log \frac{y+dy}{y}\right] = E\left[\log \frac{x+dx}{x}\right]$$

which may be abridged as

$$d\log y = d\log x. \tag{56}$$

In fact, suppose that the two ratio components y and x now are generated by the stochastic differential equations

$$d\log y_{ji} = s_j dt + \sigma_y dz_{y_i} \qquad \text{and} \qquad d\log x_{ji} = s_j dt + \sigma_x dz_{x_i} \tag{57}$$

where continuous rates of change $d\log y_{ji}$ and $d\log x_{ji}$ stem from a deterministic term, $s_j dt$, which is the same for both components and assumed to be constant throughout the process[1], plus a random term, dz_{y_i} or dz_{x_i}, specific to realisation i.[2] The summation of all dt, t, reflects the length of the accounting period during which the generation of the j^{th} financial statement takes place, typically one year.

By exponentiation, (57) leads to

$$\frac{dy_{ji}}{y_{ji}} = (s_j + \frac{\sigma_y^2}{2})dt + \sigma_y dz_{y_i} \qquad \text{and} \qquad \frac{dx_{ji}}{x_{ji}} = (s_j + \frac{\sigma_x^2}{2})dt + \sigma_x dz_{x_i}$$

which, after integration, yields

$$y_{ji} = y_0 e^{s_j t} e^{\sigma_y \mathcal{Z}_{y_i}} \qquad \text{and} \qquad x_{ji} = x_0 e^{s_j t} e^{\sigma_x \mathcal{Z}_{x_i}}$$

where y_0, x_0 are arbitrary constant magnitudes and $\mathcal{Z}_{y_j}, \mathcal{Z}_{x_j}$ are Wiener processes. Ratios of variables generated as above evolve as

$$\frac{y_{ji}}{x_{ji}} = \frac{y_0}{x_0} e^{\mathcal{Z}_i} \tag{58}$$

thus removing s_j, the random effect of size, from measurement. The term $\mathcal{Z}_i$ is also a Wiener process, with variance $(\sigma_y^2 + \sigma_x^2 - 2\,\rho\,\sigma_y\,\sigma_x)\,t$, ρ being the correlation coefficient between z_{y_i} and z_{x_i}.

Ratio components y_{ji} and x_{ji} in (58) obey (56), the robust formulation of scale invariance. In fact, it is the continuous rate of change s rather than the effective rate of change r which is expected to be similar in both components. If the two processes above were assumed to equate effective rates on average, they would be described as

$$\frac{dy_{ji}}{y_{ji}} = r_j dt + \sigma_y dz_{y_i} \qquad \text{and} \qquad \frac{dx_{ji}}{x_{ji}} = r_j dt + \sigma_x dz_{x_i}. \tag{59}$$

This is the first and most basic process tested by Tippett and Whittington (1995) for the presence of a drift term in the log of the ratio.[3] However, as shown above, continuous processes which obey the robust formulation of scale invariance lead to (58), where the log of the ratio no longer drifts either upwards or downwards.[4]

In summary, the conditions of ratio validity leading to (56) are also feasible when, as in the above models, variables are the widely used continuous-time Markov processes.

5.4 Notes

1. The continuous rate of growth $s_j = s\tau_j$ is the same for all items in the j^{th} financial statement, modelling the random effect of size upon financial statement j, irrespective of the variable considered. That is, the driving force behind the relative magnitude of the accounting numbers observed in a financial statement is s_j. For instance, firms which are larger than the industry expectation exhibit positive s_j whereas those which are smaller have negative s_j.

2. Random terms dz_{y_i}, dz_{x_i} are limits of increments of Wiener processes $\mathcal{Z}_{y_i}, \mathcal{Z}_{x_i}$ as the time interval approaches the infinitesimal dt. As mentioned, $dz_{y_i} = z_{y_i}\sqrt{dt}$ and $dz_{x_i} = z_{x_i}\sqrt{dt}$ where z_{y_i} and z_{x_i} are time-independent standard Normal random variables.

3. The distinction between the two processes (57) and (59) is seldom drawn in Financial Economics because, in general, such a distinction would be irrelevant given the context. Nevertheless, when the issue of interest is the existence or not of processes able to remove a common effect such as firm size, the distinction is essential.

4. In (58), the time dependence in the variance of z generates time dependence in the expected ratio because, as the magnitude of z increases, negative and positive realisations are differently treated by the exponentiation, spanning the intervals $\{0, 1\}$ and $\{1, \infty\}$ respectively. This

asymmetry, however, is no longer specific to ratios and a simple logarithmic transformation removes it.

Part III

Conclusions and References

Chapter 6

Conclusions

The theoretical insights and the empirical findings presented in the preceding chapters lead us to conclude that:

- The statistical distribution of financial ratios and their aggregates exhibit a multiplicative nature. Most of the aggregates are close to lognormality whereas ratios are, in general, log-leptokurtic.

- Departures from a multiplicative type of behaviour in ratios are caused by the constraining effect of the denominator over the numerator. Constant terms play a less significant role in explaining the distribution of ratios.

- Three-Parametric lognormality in financial aggregates is widespread and must be accounted for when using ratios for measurement.

- Financial ratios and the aggregates from which they are constructed may or may not obey the Markov condition. In any of these cases, however, the basic requirement for the validity of ratios, scale-invariance, may hold.

Given the multiplicative character of aggregates, it is clear that ratios are the appropriate tools for achieving comparability since they are able to remove any effect impinging upon both the numerator and the denominator. Common effects such as firm size would not be removed by ratios unless they were multiplicative. In fact, where two observations, y and x, are under the same, say, additive effect A, then y and x are described as

$$\begin{aligned} y &= A + e_y \\ x &= A + e_x, \end{aligned} \tag{60}$$

e_y and e_x being residuals. In order to remove A, y and x should be subtracted: quotients would not be effective in removing A. Firm size or any other common effect is removed by the ratio when a multiplicative effect is assumed: where two observations, y and x, are influenced by the same S (an expected proportion), then y and x are described as

$$\begin{aligned} y &= S f_y \\ x &= S f_x, \end{aligned} \tag{61}$$

f_y and f_x being random proportions of S found in y and x. In order to remove S, a ratio of y and x, not a subtraction, should be formed:

$$y - x = S(f_y - f_x) \qquad \text{whereas} \qquad \frac{y}{x} = \frac{f_y}{f_x}. \tag{62}$$

The use of ratios thus agree with the statistical nature of aggregates.

It may then be asked why some previous contributions to the literature on financial ratios have led to such a pessimistic view of ratio analysis. The reasons may be twofold. First, it is often assumed that accounting variables are, like many random variables, statistically additive. They are not. The statistical foundations of ratio analysis should be based on the understanding that accounting variables are multiplicative, being governed by proportionate or exponential growth. Second, the way in which firm size influences financial variables has also been misunderstood, leading to much uncertainty as to whether ratios remove size or not.

In this context, we return to the often-quoted statement attributed to Lev & Sunder (1979):- that almost all of the assumptions required for valid ratio analysis are likely to be violated in practice. The statement is formally correct, of course, but it might as well be applied to Newton's Laws of Motion and to many other models considered as good enough approximations in normal circumstances. Nevertheless, the statement by Lev & Sunder is misleading. Assumptions may indeed be violated but without invalidating the methodology. Distortion, in spite of its presence in mathematical models, may be small in specific cases such as in the case of non-proportionality. Moreover, when weighing inaccuracy against the ability to provide an intuitive interpretation with a parsimonious model, it may well be that such a trade-off could prove to be largely favourable to the less accurate methodology.

The kind of trade-off referred to above is particularly relevant to the ratio method. Ratios, having just one degree of freedom, are able to measure deviations from expected proportions. The condition of scale invariance is a direct consequence of this: one unique parameter is able to deal with scale invariant changes, i.e. the modelling of the common growth of both components. By providing deviations from expectations, ratios yield, in a succinct form, the information which financial analysts seek.

This thesis has uncovered a range of important questions which might be useful directions for further research. First, it would be interesting to obtain larger time-histories of firms so as to be able to get a firm basis to the results of Chapter 3. If we could use a series of, say, 30 years of accounts, other more accurate tests, such as the Unit Root test, could be used.

Another interesting question is whether, in the real world processes governing the generation of accounting aggregates obeys, or not, the scale invariance condition. In order to test this, large time-histories would also be required.

Finally, scale invariance is probably not the sole condition for the valid use of ratios. Other conditions, related to the variance of aggregates, may also be important. It would be interesting to uncover these conditions, relating them to the ever-increasing variability of Brownian motions, using extensions to the above framework.

Chapter 7

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Appendix A

Cross-Section Analysis

The tables presented in this appendix show the detailed results of the cross-section analysis carried out in Chapter 2. The first 30 tables show results of the lognormality test for each year (1982-1989) considered, and for each of the three industries (Properties, Consolidated Enterprises, and Hotels). Table 31 and 32 then illustrate the results for the same test, but for all industries together and for each year.

Log of Negative Operating Profit

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.61	0.49	-0.61	-0.15	0.6	-0.56	-1.67	-1.86
	2.42	-0.53	-0.33	0.14	-0.72	-0.16	2.92	3.49
	13	22	17	16	11	11	4	5
	0	0	0	0	0	0		
4		1.11	-1.66	-0.22			0.69	-1.76
		0.48		-3.32			-0.67	3.5
	2	6	3	4	2	1	6	6
				0			0	
6								
		1	2	2	1	1	1	1

Table 13: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Negative Operative Profit. When the absence of skewness or kurtosis occurs, it is due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Creditors

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.11	0.06	0.14	-0.03	-0.12	-0.09	-0.02	0.03
	-0.74	-0.38	-0.54	-0.35	-0.33	-0.41	-0.68	-0.58
	74	76	76	72	70	71	67	67
	0	0	0	0	0	0	0	0
4	0.31	0.47	0.03	0.12	-0.05	-0.02	-0.33	0.02
	0.18	0.1	-0.31	-0.55	-0.25	-0.22	-0.43	0.51
	35	35	36	37	37	40	43	55
	0	0	0	0	0	0	0	4000
6	-1.98	-2.15	-1.64	-1.31	-0.82	-0.51	-1.92	-1.16
	4.51	4.98	2.65	1.69	-0.71	-0.46	5.01	1.71
	10	11	11	12	13	13	11	12
					0	0		

Table 14: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Creditors. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Debtors

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.01	-0.01	-0.18	-0.38	-0.27	-0.22	-0.23	0
	-0.68	-0.75	0.12	-0.11	-0.21	-0.24	-0.65	-0.7
	72	75	75	72	70	71	67	67
	0	0	0	0	0	0	0	0
4	0.14	0.02	-0.21	0.21	-0.3	-0.09	-1.01	0.03
	0.31	-0.03	-0.44	-0.35	-0.27	0.21	4.41	1.11
	35	35	35	36	37	40	43	55
	0	0	0	0	0	0		2300
6	-1.71	-1.5	-1	-0.91	-1.19	-1.52	-1.02	0.07
	2.51	2.1	-0.47	-0.55	-0.2	2.36	0.26	-1.26
	10	11	11	12	13	13	11	12
				0				0

Table 15: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Debtors. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Net Income

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.36	-0.17	-0.34	-0.17	-0.36	-0.18	-0.11	0.14
	-0.51	-0.68	-0.29	-0.36	-0.44	0.35	-0.53	-0.86
	51	47	54	54	58	64	60	58
	0	0	0	0	0	0	0	0
4	-0.23	0.09	-0.6	-0.19	0.06	0.17	0.13	0.59
	-0.1	-0.5	0.97	-0.25	0.1	-0.77	-0.15	-0.33
	31	30	30	28	35	39	40	50
	0	0	0	0	0	0	0	0
6	-0.19	-0.32	-0.79	-0.72	0.006	-0.63	-0.01	-1.75
	-0.75	-0.05	0.58	0.55	-0.352	0.35	0.989	3.88
	9	9	8	10	13	11	11	12
	0	0	0	0	-240000	0	-25000	

Table 16: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Net Income. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Operating Profit

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.05	-0.11	-0.81	-0.75	-0.09	-0.2	-0.19	-0.1
	-0.4	-0.5	1.16	1.11	-0.65	-0.2	-0.93	0.11
	61	54	59	57	59	60	63	62
	0	0	0	0	0	0	0	0
4	0.29	0.42	-0.47	0.57	0.17	0.2	-0.03	-0.72
	-0.52	-0.28	1.28	-0.44	-0.17	-0.36	-0.25	-0.03
	33	29	33	33	35	39	37	49
	0	0	0	0	0	0	0	0
6	-0.72	-0.36	-0.52	-1	-0.55	-0.48	-0.9	-0.8
	-0.67	-0.18	0.46	1.07	-1.02	-0.62	0.49	0.55
	10	10	9	10	12	12	10	11
	0	0	0		0	0	0	0

Table 17: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Operative Profit. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Inventory

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.74	-0.3	-0.42	-0.33	-0.34	-0.43	-0.58	-0.04
	0.37	-0.71	-0.7	-0.67	0.51	0.77	0.9	-0.47
	47	49	46	46	43	48	43	47
	0	0	0	0	0	0	0	0
4	-0.4	-0.31	-0.67	-0.7	-0.9	-0.55	-0.44	-0.31
	0.29	0.04	0.44	0.23	0.68	0.06	-0.04	0.11
	30	29	29	31	32	36	38	49
	0	0	0	0	0	0	0	0
6	-0.85	-0.22	0.01	0.04	0.07	-2.08	0.02	-1.35
	0.11	0.16	-1.2	-1.16	-0.83	5.79	0.88	3.47
	7	8	8	9	10	11	9	11
	0	0	0	1700	1700		1100	

Table 18: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Inventory. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Negative Net Income

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.44	0.19	-0.38	-0.18	-0.25	0.37	0.52	-0.65
	-0.45	-0.7	-0.43	-1.2	-1.43	-1.44	-0.66	-0.09
	23	29	22	19	12	7	7	9
	0	0	0	0	0	0	0	0
4	1.28	-0.41	0.65	1.41			0.53	-0.57
	1.88	-2.99	-1.68	1.69				-0.23
	4	5	6	9	2	1	3	5
		0	0				0	0
6			-1.47					
	1	2	3	2	1	2	1	

Table 19: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Negative Net Income. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Negative Working Capital

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.21	0.14	-0.35	-0.16	-0.24	-0.31	-0.28	-0.29
	0.38	-0.18	-0.05	0.85	1.61	-0.24	-1.16	-0.4
	24	22	26	23	24	22	27	27
	0	0	0	0	0	0	0	0
4	-1.42	-0.79	0.45	-0.13	0.97	-0.26	-1.25	-0.44
	2.07	-1.02	-1.78	1.14	0.26	-0.45	1	-1.97
	12	14	9	11	9	10	12	10
		0	0	0	0	0		0
6	-1.45	-0.02	0.18	-0.35	-1.23	-0.05	2.22	0.26
		1.05	-1.43	1.46	2.39	-0.82	5.13	-0.03
	3	4	7	7	9	9	6	4
		0	0	0		0		0

Table 20: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Negative Working Capital. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Working Capital

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.11	-0.51	-0.08	0.26	-0.41	0.37	-0.07	0.43
	-0.46	0.87	-0.44	-0.56	1.22	0.25	0.72	0.04
	50	54	50	50	46	49	40	40
	0	0	0	0	0	0	0	0
4	-0.15	0.04	-0.52	0.25	-0.08	-0.62	0.29	0.16
	0.61	-0.82	0.39	-0.63	-0.19	1.88	0.56	0.22
	23	21	27	26	28	30	31	45
	0	0	0	0	0	0	0	0
6	-0.55	-1.88	0.97	-0.12	0.54	-0.75	-0.07	-0.34
	0.34	4.06	1.31	-2.4	-2.87	-1.31	-1.92	-1.44
	7	7	4	5	4	4	5	8
	0		0	0	0	0	0	0

Table 21: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Working Capital. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Expenses

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.22	0.07	-0.42	-0.2	-0.48	-0.4	-0.24	-0.05
	-0.29	-0.18	0.54	-0.05	-0.35	0.3	-0.49	-0.82
	60	64	68	67	66	69	66	66
	0	0	0	0	0	0	0	0
4	-0.37	-0.38	-0.21	-0.19	-0.25	0.15	0.23	0.12
	0.98	1	-0.27	-0.31	-0.06	-0.38	-0.57	0.36
	31	30	33	33	35	37	41	53
	0	0	0	0	0	0	0	0
6	-1	-1.32	-2.04	-1.75	-1.56	-0.45	-0.95	-1.76
	0.66	0.46	4.41	3.1	2.46	0.5	1.01	3.58
	9	10	10	12	12	13	11	12
						0	0	

Table 22: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Expenses. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Current Liabilities

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.2	-0.37	-0.42	-0.65	-0.68	-0.57	0.15	-0.18
	0.27	0.35	0.49	0.44	0.24	0.67	-1.05	-0.34
	74	76	76	73	70	71	67	67
	0	0	0	0	0	0	0	0
4	0.48	0.35	0.03	-0.22	-0.21	0.02	0.46	0.57
	-0.39	-0.35	-0.38	0.17	-0.12	-0.58	-0.47	0.03
	35	35	36	37	37	40	43	55
	0	0	0	0	0	0	0	0
6	-1.27	-1.53	-1.16	-0.9	-1.49	0.11	0.56	-1.11
	1.71	2.17	1.41	0.14	1.78	-0.94	0.04	1.4
	10	11	11	12	13	13	11	12
				0		0	0	

Table 23: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Current Liabilities. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Sales

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.25	-0.65	-0.3	-0.38	-0.58	-0.91	-0.09	0.1
	-0.16	2.06	0.42	0.36	0.29	2.59	-0.74	-0.84
	63	68	70	68	68	69	66	66
	0	0	0	0	0	0	0	0
4	0.24	0.32	-0.1	-0.61	-0.13	0.27	0.2	-0.07
	0.29	-0.06	-0.48	0.75	-0.16	-0.49	-0.58	1.09
	32	33	34	35	36	38	41	54
	0	0	0	0	0	0	0	0
6	-0.8	-1.05	-1.31	-1.36	-1.1	-0.29	-1.3	-0.79
	0.38	0.58	1.07	0.92	0.43	-0.67	2.13	-0.03
	9	10	11	12	13	13	11	12
	0					0		0

Table 24: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Sales. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Total Debts

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.09	-0.19	-0.28	-0.62	-0.66	-0.62	0.02	-0.28
	0.24	0.43	0.6	0.47	0.43	0.62	-1.08	-0.39
	74	76	76	73	70	71	67	67
	0	0	0	0	0	0	0	0
4	0.51	0.33	0.08	-0.21	-0.23	0.06	0.49	0.04
	-0.39	-0.32	-0.29	0.14	-0.08	-0.5	-0.42	1.3
	35	35	36	37	37	40	43	55
	0	0	0	0	0	0	0	9000
6	-0.68	-1.02	-0.95	-0.75	-1.31	-0.26	0.12	-0.88
	0.84	1.05	0.71	0.3	1.01	-1.66	-1.23	0.65
	10	11	11	12	13	13	11	12
	0		0	0		0	0	0

Table 25: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Total Debts. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Fixed Assets

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	-0.16	-0.01	0.03	-0.05	0.06	0.08	0.3	0.07
	-0.4	-0.43	-0.42	-0.44	-0.62	-0.96	-0.44	-0.62
	74	76	76	73	70	71	61	63
	-1000	-1000	-1000	-1000	-1000	-500	0	0
4	0	0	-0.06	-0.1	-0.06	-0.84	0	-0.93
	0.31	0.31	0.24	0.58	-0.13	1.58	-0.67	3.22
	35	35	36	36	37	40	43	55
	-200	-550	-500	0	-3000	0	-3300	0
6	-0.5	-1.15	-0.96	-0.29	-0.09	-0.61	0.06	-0.38
	-0.91	0.37	0.38	-0.38	-1.11	0.9	-1.55	-1.38
	10	11	11	11	13	13	10	11
	0		0	0	-10000	0	0	0

Table 26: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Fixed Assets. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Debts

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.05	0.15	-0.03	0.08	0.43	-0.37	0.05	-0.3
	-0.34	-0.18	-0.02	-0.35	-0.73	-0.16	-1.85	-0.81
	30	31	31	31	26	31	67	45
	0	0	0	0	0	0	-1	0
4	0.23	0.23	-0.01	-0.4	0.03	0.03	0.08	0
	-0.85	-0.34	-0.36	0.32	-1.67	-1.77	-1.77	-1.28
	18	18	18	18	37	40	43	55
	0	0	0	0	-10	-1	-1	-20
6		-1.3	0.07	-1.55	-0.01	0.01	0.01	0.01
			-4.92	2.46	-2.21	-1.65	-1.98	-1.52
	2	3	4	4	13	13	11	12
			0		-10	-100000	-7000	-12000

Table 27: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Debts. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Current Assets

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.7	-0.61	-0.78	-0.64	-0.59	-0.45	-0.44	0.11
	1.35	0.89	1.13	0.7	0.77	0.8	1.26	-0.45
	74	76	76	73	70	71	67	67
	0	0	0	0	0	0	0	0
4	0.47	-0.11	-0.01	0.06	-0.17	0.15	0.32	0.38
	-0.31	-0.01	-0.21	-0.49	-0.21	-0.72	-0.66	-0.03
	35	35	36	37	37	40	43	55
	0	0	0	0	0	0	0	0
6	-1.02	-1.52	-0.77	-0.12	-0.83	0.11	-1.2	-0.58
	0.5	1.24	-0.32	-0.32	0.92	-1.02	1.87	0.21
	10	11	11	12	13	13	11	12
			0	0	0	0		0

Table 28: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Current Assets. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Net Worth

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.34	0.02	0.09	-0.4	-0.04	0.15	-0.03	0.06
	-0.14	0.73	0.54	1.91	0.16	0.37	0.95	-0.08
	74	75	75	72	69	71	67	67
	0	0	0	0	0	0	18000	34500
4	-0.01	0.03	-0.03	0.01	0	-0.01	0.02	0.06
	0.34	0.62	-0.05	-0.09	-0.02	-0.05	1.12	1.11
	35	35	36	36	36	40	43	55
	15000	16000	17000	16000	16000	18500	7500	46500
6	-0.17	-0.14	-0.48	-0.37	-1.11	-1.97	-0.35	-0.68
	0.16	0.51	0.44	-0.76	0.11	4.62	-1.2	-0.48
	10	10	10	12	12	13	11	12
	0	0	0	0			0	0

Table 29: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Net Worth. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

Log of Total Assets

Ind	1982	1983	1984	1985	1986	1987	1988	1989
3	0.4	0.17	0.19	-0.1	-0.13	0.02	0.51	0.48
	-0.12	0.19	0.09	0.33	0.28	0.35	-0.69	-0.78
	74	76	76	73	70	71	67	67
	0	0	0	0	0	0	0	0
4	0.04	0.01	0.01	0.29	0.23	0.26	0.02	-0.02
	0.42	0.3	-0.24	-0.55	-0.53	-0.63	0.93	1.71
	35	35	36	37	37	40	43	55
	17500	18500	17500	0	0	0	13500	56000
6	-0.34	-0.82	-0.88	-0.58	-0.89	-0.81	-0.48	-0.45
	-0.16	0.7	0.36	-0.01	-0.43	-0.1	-1.34	-0.99
	10	11	11	12	13	13	11	12
	0	0	0	0	0	0	0	0

Table 30: For each industry and year, the table shows the skewness, the kurtosis, number of cases and the delta (if any) for the variable, log of Total Assets. In the cases of absence of skewness or kurtosis. it is because they cannot be obtained due to the insufficient number of cases or no data available. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the variable.

All Industries in the Same Sample

	Year							
Var.	1982	1983	1984	1985	1986	1987	1988	1989
S	-0.37	-0.62	-0.39	-0.48	-0.68	-0.85	-0.1	-0.08
	0.22	1.96	0.35	0.31	0.63	2.74	-0.25	-0.19
	104	111	115	115	117	120	118	132
	0		0	0	0		0	0
D	-0.09	-0.05	0.19	-0.39	-0.37	-0.39	-0.44	-0.19
	-0.24	-0.41	-0.1	0.2	-0.11	0.07	0.33	0.06
	117	121	121	120	120	124	121	134
	0	0	0	0	0	0	0	0
I	-0.73	-0.38	-0.62	-0.41	-0.41	-0.59	-0.67	-0.31
	0.81	-0.25	-0.08	-0.35	0.39	0.83	1.3	-0.03
	84	86	83	86	85	95	90	107
	0	0	0	0	0	0		0
CA	-0.59	-0.48	-0.61	-0.54	-0.61	-0.32	-0.3	0.09
	1.43	0.8	1.17	0.83	0.92	0.76	1.02	-0.14
	119	122	123	122	120	124	121	134
		0		0	0	0		0
FA	-0.47	-0.59	-0.53	-0.44	-0.00	-0.63	-0.2	-0.4
	1.75	1.75	1.28	1	-0.64	0.19	-0.71	-0.36
	113	115	116	114	120	117	113	129
					-1700	0	0	0
TA	0.36	0.15	0.14	-0.11	-0.21	-0.02	0.41	0.49
	0.2	0.11	-0.06	0.29	0.21	0.18	-0.48	-0.48
	119	122	123	122	120	124	121	134
	0	0	0	0	0	0	0	0
C	-0.06	-0.05	0.02	-0.07	-0.2	-0.11	-0.07	-0.12
	-0.22	0.12	-0.35	-0.19	-0.17	-0.12	-0.17	0.07
	119	122	123	121	120	124	121	134
	0	0	0	0	0	0	0	0
CL	-0.19	-0.31	-0.35	-0.62	-0.71	-0.57	0.09	-0.16
	0.41	0.5	0.43	0.67	0.65	1.01	-0.58	0.24
	119	122	123	122	120	124	121	134
	0	0	0	0	0	0	0	0
DB	0.11	0.07	-0.15	-0.13	0.32	0	-0.3	0.01
	-0.54	-0.35	-0.16	-0.42	-0.75	-0.27	0.26	-0.83
	50	52	53	53	54	63	67	91
	0	0	0	0	0	0	0	0

Table 31: This table shows the results of normality tests in the case where all industries are pulled together in the same sample. For each variable and year, the table displays the skewness, the kurtosis, the number of cases and delta (if any). Displayed values refer to the transformed variable: $T = \log(V - \delta)$. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the log of the variable.

All Industries in the Same Sample

	Year							
Var.	1982	1983	1984	1985	1986	1987	1988	1989
NW	0.3	0.04	0.08	-0.33	-0.14	-0.04	0.47	0.56
	-0.15	0.68	0.37	1.73	0.19	0.32	-0.33	-0.55
	119	120	121	120	117	124	121	134
	0	0	0		0	0	0	0
OP	-0.09	-0.15	-0.77	-0.62	-0.19	-0.37	-0.08	-0.21
	-0.34	-0.15	1.37	1.36	-0.42	0.27	-0.68	0.61
	104	93	101	100	106	111	110	122
	0	0			0	0	0	0
NI	0.22	-0.12	-0.4	-0.1	-0.39	-0.22	-0.29	-0.01
	-0.46	-0.38	0.04	-0.35	-0.04	0.34	-0.13	-0.21
	91	86	92	92	105	114	110	120
	0	0	0	0	0	0	0	0
WC	-0.01	-0.52	-0.23	0.19	-0.49	-0.04	-0.1	0.11
	-0.3	1.04	0	-0.56	1.14	0.32	0.5	-0.06
	80	82	81	81	78	83	76	93
	0		0	0		0	0	0
EX	-0.35	-0.27	-0.54	-0.4	-0.62	-0.42	-0.2	-0.24
	-0.05	-0.09	0.54	0.06	0.15	0.59	0.03	-0.15
	100	104	111	112	113	119	118	131
	0	0	0	0	0	0	0	0
TD	-0.07	-0.18	-0.22	-0.59	-0.7	-0.65	0	-0.23
	0.28	0.38	0.38	0.63	0.67	1.07	-0.58	0.24
	119	122	122	122	120	124	121	134
	0	0	0	0	0		0	0
Ng OP	0.57	0.39	-0.76	-0.59	0.14	-0.7	0.4	-1.45
	2.56	-0.76	0.05	0.09	-1.41	-0.23	1.32	1.19
	15	29	22	22	14	13	11	12
		0	0	0		0		
Ng NI	-0.27	0.03	-0.12	-0.14	-0.26	-0.4	0.44	-0.53
	-0.85	-0.89	-0.35	-0.11	-1.03	-1.62	-0.41	-0.6
	28	36	31	30	15	10	11	14
	0	0	0	0			0	0
Ng WC	-0.37	-0.33	-0.7	-0.27	-0.19	-0.37	-0.2	-0.18
	-0.04	-0.57	0.55	0.77	1.75	0.3	-0.76	-0.72
	39	40	42	41	42	41	45	41
	0	0	0	0		0	0	0

Table 32: This table shows the results of normality tests in the case where all industries are pulled together in the same sample. For each variable and year, the table displays the skewness, the kurtosis, the number of cases and delta (if any). Displayed values refer to the transformed variable: $T = \log(V - \delta)$. When no delta is presented, this is an indication that the test has failed to achieve significance, denoting non-normality of the log of the variable.

Appendix B

Time-Series Analysis

This appendix contains all the tables used for the time-series analysis. We start by listing the tables for the unconstrained variables for each industry and for each year. Then the same applies to the tables for the constrained ratios. After that a detailed table for calculating zeta is presented and finally the regression tables.

Industry: 3		Year: 1982
Variable	Kurtosis	Skewness
OP/S	12.03	-3.09
C/TA	12.43	2.97
I/TA	3.68	1.92
CL/TA	3.32	1.57
C/CL	1.73	1.38
TD/TA	1.56	1.19
FA/TA	-.70	.62
CA/TA	1.35	1.02
I/CA	-1.14	.33
NW/TA	1.10	-1.08

Table 33: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3		Year: 1983
Variable	Kurtosis	Skewness
OP/S	29.23	-5.20
C/TA	25.35	4.33
I/TA	1.84	1.54
CL/TA	7.32	2.22
C/CL	1.31	1.46
FA/TA	-.95	.56
TD/TA	3.03	1.43
CA/TA	-.31	.58
I/CA	-1.31	.27
NW/TA	-.25	-.73

Table 34: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3		Year: 1984
Variable	Kurtosis	Skewness
OP/S	31.98	-5.32
C/TA	26.10	4.37
I/TA	2.48	1.60
CL/TA	8.67	2.37
C/CL	1.31	1.44
FA/TA	-.94	.46
TD/TA	5.68	1.78
CA/TA	.29	.68
I/CA	-1.46	.17
NW/TA	-.29	-.70

Table 35: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3 Year: 1985

Variable	Kurtosis	Skewness
C/TA	10.74	2.86
I/TA	.71	1.23
OP/S	2.58	.18
CL/TA	14.59	3.19
C/CL	.67	1.21
TD/TA	9.33	2.35
FA/TA	-.74	.62
I/CA	-.93	.46
CA/TA	.23	.77
NW/TA	.71	-.91

Table 36: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3 Year: 1986

Variable	Kurtosis	Skewness
C/TA	10.54	2.87
OP/S	16.44	-3.87
I/TA	7.63	2.40
CL/TA	11.50	2.85
TD/TA	7.80	2.19
FA/TA	-.10	1.06
I/CA	-.82	.75
C/CL	.28	1.10
CA/TA	-.55	.59
NW/TA	.24	-.86

Table 37: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3 Year: 1987

Variable	Kurtosis	Skewness
OP/S	65.20	-7.98
C/TA	3.60	1.94
I/TA	10.21	2.81
CL/TA	5.99	2.14
FA/TA	2.16	1.69
TD/TA	.70	1.00
I/CA	.30	1.17
C/CL	.74	1.17
CA/TA	-.38	.68
NW/TA	.50	-.88

Table 38: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3	Year: 1988	
Variable	Kurtosis	Skewness
C/TA	6.55	2.45
I/TA	15.53	3.61
FA/TA	8.06	2.93
CL/TA	2.35	1.45
TD/TA	1.53	1.05
I/CA	.30	1.06
CA/TA	.82	1.15
C/CL	1.56	1.42
OP/S	-.86	.27
NW/TA	1.44	-.99

Table 39: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3	Year: 1989	
Variable	Kurtosis	Skewness
C/TA	8.12	2.67
FA/TA	23.33	4.72
I/TA	20.97	4.06
CL/TA	1.17	1.01
I/CA	2.09	1.34
CA/TA	1.71	1.28
C/CL	1.01	1.40
TD/TA	1.80	1.07
OP/S	-.18	-.02
NW/TA	1.80	-1.06

Table 40: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4	Year: 1982	
Variable	Kurtosis	Skewness
C/TA	18.67	3.89
I/TA	1.10	1.21
OP/S	15.13	3.48
CL/TA	3.67	1.80
I/CA	.36	.89
FA/TA	-1.23	.24
TD/TA	-.22	.83
C/CL	-.66	.67
CA/TA	-1.05	.20
NW/TA	-.35	-.75

Table 41: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1983
Variable	Kurtosis	Skewness
C/TA	15.19	3.53
I/TA	-.15	1.06
I/CA	-.86	.73
OP/S	11.72	2.93
CL/TA	1.44	1.09
FA/TA	-.10	.75
C/CL	-.53	.67
TD/TA	-.53	.50
CA/TA	-.79	.10
NW/TA	-.82	-.30

Table 42: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1984
Variable	Kurtosis	Skewness
I/TA	-1.06	.50
C/TA	8.70	2.86
OP/S	6.22	-.34
I/CA	-1.25	.36
CL/TA	1.69	1.28
FA/TA	-.56	.52
TD/TA	-.28	.66
C/CL	-.47	.54
CA/TA	-.59	.26
NW/TA	-.88	-.48

Table 43: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1985
Variable	Kurtosis	Skewness
OP/S	29.70	-5.17
I/TA	1.26	1.25
C/TA	8.36	2.81
I/CA	-.98	.49
FA/TA	-1.10	.42
CL/TA	.22	.79
C/CL	-.64	.56
TD/TA	.27	.71
CA/TA	-.79	.32
NW/TA	-.84	-.19

Table 44: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1986
Variable	Kurtosis	Skewness
C/TA	11.01	2.87
I/TA	.72	1.08
OP/S	15.57	3.27
I/CA	-1.18	.46
FA/TA	-1.01	.59
CL/TA	.20	.77
C/CL	-.44	.37
TD/TA	1.91	1.01
CA/TA	-1.14	.15
NW/TA	-.64	-.07

Table 45: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1987
Variable	Kurtosis	Skewness
I/TA	-.59	.88
C/TA	11.64	3.01
OP/S	12.11	2.98
I/CA	-.69	.65
FA/TA	-.91	.57
CL/TA	2.56	1.39
C/CL	-.69	.46
TD/TA	.86	.85
CA/TA	-1.26	.20
NW/TA	.56	-.68

Table 46: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1988
Variable	Kurtosis	Skewness
I/TA	-.14	.99
C/TA	1.71	1.43
OP/S	.26	.38
FA/TA	1.10	1.21
I/CA	-1.25	.41
CL/TA	.67	.69
TD/TA	.53	.52
C/CL	-.40	.52
CA/TA	-.65	.53
NW/TA	.33	-.42

Table 47: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 4		Year: 1989
Variable	Kurtosis	Skewness
C/TA	.50	1.10
OP/S	21.17	-2.89
I/TA	.58	1.11
FA/TA	2.02	1.43
CL/TA	-1.03	.19
I/CA	-.74	.65
TD/TA	-.31	.34
C/CL	-1.06	.23
CA/TA	-.73	.36
NW/TA	-.26	-.38

Table 48: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6		Year: 1982
Variable	Kurtosis	Skewness
I/TA	2.59	1.64
C/TA	-.37	.77
I/CA	-.72	1.04
CA/TA	4.93	2.01
CL/TA	-1.02	.60
C/CL	-1.62	.29
TD/TA	-.43	.63
OP/S	-.51	.03
FA/TA	-.81	.59
NW/TA	-.82	-.49

Table 49: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6		Year: 1983
Variable	Kurtosis	Skewness
I/TA	6.61	2.52
C/TA	-1.08	.30
I/CA	2.83	1.74
OP/S	7.00	-2.45
CL/TA	-.18	.71
CA/TA	3.04	1.56
C/CL	-.54	-.05
TD/TA	1.20	1.22
FA/TA	-.80	.39
NW/TA	.06	-.75

Table 50: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6 Year: 1984

Variable	Kurtosis	Skewness
I/TA	-.39	1.15
C/TA	-.77	.40
C/CL	.74	.81
CA/TA	2.22	1.59
I/CA	-.64	.90
CL/TA	2.23	1.47
TD/TA	6.26	2.30
FA/TA	-1.46	.39
OP/S	7.64	2.60
NW/TA	-1.47	-.19

Table 51: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6 Year: 1985

Variable	Kurtosis	Skewness
OP/S	11.52	-3.37
I/TA	4.66	1.94
C/TA	5.07	2.11
I/CA	.45	1.11
CA/TA	.24	.97
CL/TA	4.60	1.94
C/CL	-.20	.11
TD/TA	1.92	1.24
FA/TA	-1.34	.20
NW/TA	3.03	-1.52

Table 52: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6 Year: 1986

Variable	Kurtosis	Skewness
I/TA	7.98	2.76
C/TA	10.84	3.23
I/CA	.82	1.04
CA/TA	5.58	2.28
CL/TA	11.60	3.35
OP/S	1.97	.29
C/CL	4.26	1.85
TD/TA	9.86	2.98
FA/TA	-1.79	.04
NW/TA	-1.14	.04

Table 53: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6 Year: 1987

Variable	Kurtosis	Skewness
I/TA	9.58	3.05
C/TA	9.86	3.07
I/CA	3.53	1.76
CA/TA	3.94	2.07
CL/TA	10.59	3.16
OP/S	5.93	-2.18
C/CL	-.57	.19
TD/TA	2.59	1.48
FA/TA	-1.66	.03
NW/TA	1.97	-1.32

Table 54: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6 Year: 1988

Variable	Kurtosis	Skewness
I/TA	2.58	1.87
C/TA	1.67	1.46
I/CA	7.49	2.66
CL/TA	7.84	2.74
CA/TA	9.21	2.97
TD/TA	6.92	2.44
C/CL	2.04	-1.24
OP/S	1.87	-1.26
FA/TA	-1.49	.11
NW/TA	7.21	-2.48

Table 55: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 6 Year: 1989

Variable	Kurtosis	Skewness
I/TA	1.62	1.14
C/TA	-1.09	.15
I/CA	9.26	2.97
CL/TA	7.12	2.41
TD/TA	.41	.53
CA/TA	10.52	3.18
C/CL	-.77	.42
OP/S	4.72	1.46
FA/TA	-1.54	-.37
NW/TA	-.17	-.57

Table 56: The table shows the skewness and the kurtosis of the constrained ratios for the indicated industry and year.

Industry: 3		Year: 1982
Variable	Kurtosis	Skewness
OPR_GR	34.00	-4.96
STK_RT	1.01	.39
SAL_GR	3.45	-.97
ROE	19.53	3.98
S/TA	16.46	3.77
S/NW	28.41	4.98
I/S	6.87	2.54
C/D	10.72	3.25
FA/S	12.37	3.37
D/C	66.68	7.99
S/FA	41.81	6.17
CA/CL	30.88	5.48
I/C	3.58	2.03
I/D	11.19	3.28
CL/CA	63.72	7.82
NW/S	36.63	5.98
TA/S	32.32	5.68
C/I	20.80	4.68
D/I	22.08	4.80
S/I	40.82	6.35

Table 57: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1983

Variable	Kurtosis	Skewness
OPR_GR	57.69	-7.20
ROE	15.12	1.81
STK_RT	-.49	-.29
SAL_GR	3.73	-.65
S/TA	49.77	6.70
S/NW	30.44	5.21
I/S	1.77	1.56
CL/CA	47.89	6.74
S/FA	16.16	3.88
FA/S	23.69	4.55
CA/CL	30.21	5.20
C/D	47.03	6.51
D/C	39.81	6.24
I/C	6.98	2.57
C/I	20.19	4.29
D/I	40.45	6.18
I/D	46.66	6.76
NW/S	65.58	8.06
TA/S	65.79	8.05
S/I	19.52	4.27

Table 58: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1984

Variable	Kurtosis	Skewness
ROE	8.08	1.54
SAL_GR	4.66	.12
STK_RT	-.10	-.15
S/TA	7.55	2.60
S/NW	14.94	3.68
I/S	33.38	5.49
D/C	49.09	6.78
FA/S	10.67	3.03
OPR_GR	68.97	8.17
CA/CL	28.62	5.13
CL/CA	25.71	4.99
C/D	64.10	7.78
I/C	13.18	3.20
NW/S	31.95	5.17
TA/S	23.52	4.37
I/D	25.74	4.72
D/I	18.25	4.29
C/I	16.17	4.04
S/FA	64.49	8.02
S/I	23.04	4.71

Table 59: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1985

Variable	Kurtosis	Skewness
ROE	37.04	5.30
SAL_GR	1.54	-.40
STK_RT	2.34	.47
S/TA	4.72	2.08
S/NW	2.41	1.55
I/S	8.11	2.71
C/I	41.98	6.34
D/C	18.38	4.18
D/I	18.46	4.28
FA/S	18.63	3.89
CL/CA	33.00	5.52
C/D	54.37	7.09
I/C	20.93	4.08
NW/S	57.91	7.38
I/D	17.78	3.92
CA/CL	57.80	7.36
TA/S	53.53	6.96
S/FA	20.13	4.55
OPR_GR	69.15	8.23
S/I	28.18	5.07

Table 60: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1986

Variable	Kurtosis	Skewness
SAL_GR	1.84	0.50
S/TA	40.71	5.84
STK_RT	2.00	1.30
ROE	15.32	3.86
I/S	6.21	2.42
S/NW	31.84	5.05
FA/S	6.99	2.58
I/C	8.01	2.70
D/C	65.77	8.01
CL/CA	31.57	5.44
NW/S	37.81	5.55
OPR_GR	34.61	5.52
CA/CL	12.75	3.50
I/D	17.08	3.79
TA/S	32.92	5.02
C/D	27.15	4.99
D/I	42.99	6.56
S/FA	31.40	5.51
C/I	42.99	6.56
S/I	42.99	6.56

Table 61: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1987

Variable	Kurtosis	Skewness
STK_RT	3.48	1.13
SAL_GR	3.53	1.62
I/S	13.70	3.35
S/TA	31.92	5.11
S/NW	21.38	4.33
D/C	19.89	3.80
FA/S	15.59	3.81
ROE	37.04	5.96
CL/CA	35.11	5.83
I/C	44.23	6.54
C/D	29.35	5.36
I/D	22.22	4.39
CA/CL	27.36	5.06
NW/S	36.54	5.68
TA/S	35.01	5.49
OPR_GR	22.82	4.70
D/I	44.11	6.56
S/FA	57.27	7.43
C/I	47.61	6.89
S/I	47.99	6.93

Table 62: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1988

Variable	Kurtosis	Skewness
SAL_GR	1.78	.34
S/TA	5.06	2.22
STK_RT	1.28	.82
S/NW	7.69	2.70
I/S	16.98	3.71
ROE	33.12	5.34
FA/S	56.95	7.37
D/C	64.27	7.95
I/C	14.49	3.51
CA/CL	28.94	4.91
CL/CA	53.33	7.03
OPR_GR	23.38	4.69
C/D	46.77	6.51
I/D	24.77	4.73
NW/S	9.01	2.75
TA/S	7.15	2.40
S/FA	58.06	7.57
D/I	43.00	6.56
C/I	42.99	6.56
S/I	43.00	6.56

Table 63: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 3 Year: 1989

Variable	Kurtosis	Skewness
S/TA	13.19	3.35
STK_RT	3.94	1.51
SAL_GR	.90	.64
S/NW	20.06	4.22
FA/S	38.53	5.82
ROE	14.85	3.45
I/S	4.25	2.15
OPR_GR	9.63	2.66
D/C	4.83	2.29
CL/CA	38.19	5.93
C/D	6.69	2.56
I/C	38.74	6.00
I/D	4.99	2.36
CA/CL	62.00	7.77
NW/S	20.04	4.11
TA/S	21.53	4.33
D/I	21.68	4.73
C/I	41.64	6.33
S/I	23.28	4.68
S/FA	49.08	6.80

Table 64: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1982

Variable	Kurtosis	Skewness
STK_RT	.06	.27
SAL_GR	3.94	-.34
ROE	29.14	5.24
OPR_GR	7.56	1.90
I/S	6.25	2.30
S/TA	21.36	4.30
FA/S	15.90	3.65
CL/CA	8.98	2.94
D/C	1.82	1.69
S/NW	14.97	3.49
CA/CL	4.43	1.89
NW/S	7.15	2.66
I/C	2.89	1.81
C/D	1.55	1.60
I/D	16.14	3.80
D/I	22.32	4.53
TA/S	25.95	4.94
C/I	6.61	2.70
S/FA	16.02	3.88
S/I	4.23	2.12

Table 65: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1983

Variable	Kurtosis	Skewness
SAL_GR	15.06	2.80
STK_RT	16.09	3.43
ROE	32.13	5.56
I/S	1.91	1.52
S/TA	12.44	3.21
D/C	2.54	1.76
FA/S	22.65	4.48
OPR_GR	33.77	5.77
S/NW	8.56	2.74
CA/CL	3.42	1.85
I/C	1.87	1.65
I/D	12.12	3.23
NW/S	7.28	2.74
C/D	33.65	5.76
TA/S	22.09	4.55
S/FA	25.08	4.86
CL/CA	34.62	5.87
S/I	4.53	2.24
D/I	28.97	5.38
C/I	28.91	5.37

Table 66: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1984

Variable	Kurtosis	Skewness
SAL_GR	11.46	1.53
I/S	.03	.96
STK_RT	9.20	1.97
ROE	13.30	3.60
CL/CA	1.64	1.47
OPR_GR	3.46	.86
S/TA	7.88	2.74
D/C	3.67	1.73
FA/S	7.97	2.69
CA/CL	6.24	2.31
I/C	6.54	2.63
S/NW	26.49	4.95
I/D	14.77	3.47
NW/S	13.87	3.55
TA/S	18.71	4.14
C/D	31.72	5.55
S/I	10.03	2.92
D/I	29.91	5.46
S/FA	12.07	3.48
C/I	28.98	5.38

Table 67: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1985

Variable	Kurtosis	Skewness
I/S	1.49	1.42
SAL_GR	6.52	2.31
STK_RT	3.93	.44
ROE	11.99	3.46
CL/CA	17.57	3.72
S/TA	6.51	2.57
FA/S	10.38	3.28
D/C	10.70	3.19
C/D	3.73	1.81
CA/CL	10.28	2.88
S/NW	24.00	4.64
I/D	4.67	2.13
NW/S	19.47	4.31
I/C	29.46	5.37
TA/S	11.77	3.50
OPR_GR	33.24	5.66
D/I	31.03	5.54
S/FA	14.89	3.93
S/I	7.14	2.66
C/I	30.89	5.55

Table 68: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1986

Variable	Kurtosis	Skewness
I/S	1.18	1.39
ROE	10.85	3.00
STK_RT	.03	.38
SAL_GR	15.50	3.49
FA/S	28.30	5.11
CL/CA	18.39	3.97
D/C	4.37	2.05
S/TA	8.80	2.90
NW/S	11.87	3.43
CA/CL	7.39	2.70
C/D	6.83	2.31
S/NW	8.38	2.80
I/C	12.95	3.59
I/D	5.20	2.12
OPR_GR	35.30	5.88
TA/S	33.25	5.69
S/FA	23.89	4.60
D/I	31.70	5.62
S/I	6.95	2.83
C/I	31.52	5.60

Table 69: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1987

Variable	Kurtosis	Skewness
I/S	5.35	2.13
STK_RT	2.69	-.29
SAL_GR	9.33	-1.68
ROE	1.02	1.33
OPR_GR	5.91	-1.43
FA/S	16.09	3.70
CL/CA	2.56	1.42
S/TA	14.37	3.30
I/C	3.52	1.93
NW/S	8.15	2.79
CA/CL	13.89	3.25
C/D	8.94	2.80
D/C	31.29	5.38
S/NW	7.78	2.63
I/D	19.55	4.03
TA/S	33.01	5.61
D/I	35.80	5.98
S/FA	19.76	4.43
S/I	9.00	2.97
C/I	35.92	5.99

Table 70: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1988

Variable	Kurtosis	Skewness
SAL_GR	2.57	-.76
I/S	1.27	1.26
STK_RT	8.79	-1.59
ROE	8.44	2.70
FA/S	.64	1.23
OPR_GR	4.40	1.74
S/TA	22.52	4.32
CL/CA	8.11	2.90
D/C	22.10	4.32
I/C	12.32	3.16
S/NW	15.36	3.46
NW/S	5.85	2.44
CA/CL	3.15	1.80
I/D	12.08	2.99
TA/S	5.15	2.32
D/I	8.76	2.73
C/I	24.96	4.66
C/D	42.94	6.55
S/FA	12.81	3.59
S/I	6.02	2.54

Table 71: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 4 Year: 1989

Variable	Kurtosis	Skewness
OPR_GR	12.32	-2.65
STK_RT	3.15	1.33
SAL_GR	1.17	.20
I/S	21.85	4.10
ROE	12.06	3.07
FA/S	28.18	4.93
S/TA	26.95	4.58
CL/CA	39.39	6.02
D/C	4.80	2.03
C/D	7.44	2.67
S/NW	9.40	2.60
I/C	8.70	2.59
CA/CL	9.73	2.89
I/D	4.45	2.03
NW/S	10.69	3.06
D/I	5.44	2.41
C/I	4.47	2.18
TA/S	9.48	2.88
S/I	16.20	3.69
S/FA	53.72	7.32

Table 72: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6 Year: 1982

Variable	Kurtosis	Skewness
STK_RT	.72	-.86
I/S	5.28	2.27
SAL_GR	3.41	1.36
OPR_GR	-.59	.38
I/C	.55	.88
ROE	8.84	2.94
I/D	2.57	1.47
S/TA	3.02	1.68
S/NW	1.13	1.47
D/C	-.97	.94
S/FA	-.47	.87
CA/CL	-.86	.16
FA/S	-1.07	.78
NW/S	.85	.72
C/D	9.17	2.99
TA/S	-1.40	.28
D/I	-.66	1.00
CL/CA	9.43	3.05
C/I	5.58	2.31
S/I	-.97	.63

Table 73: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6		Year: 1983
Variable	Kurtosis	Skewness
OPR_GR	10.67	-3.25
STK_RT	1.36	-.28
I/S	6.93	2.63
ROE	6.40	1.97
SAL_GR	6.87	2.45
I/D	2.04	1.27
S/TA	1.41	1.27
I/C	7.53	2.72
S/NW	-.72	.51
S/FA	3.19	1.74
CA/CL	.52	.96
D/C	.46	1.29
C/D	-.48	.99
FA/S	-1.55	.30
NW/S	-.66	.99
CL/CA	10.34	3.19
D/I	-1.04	.81
TA/S	.40	.88
C/I	5.59	2.29
S/I	-.75	.63

Table 74: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6	Year: 1984	
Variable	Kurtosis	Skewness
OPR_GR	4.05	-2.08
SAL_GR	2.38	.55
I/S	2.33	1.74
STK_RT	.16	-.14
S/TA	.30	1.19
I/C	-1.98	.21
S/NW	.45	1.18
ROE	9.79	3.12
D/C	.94	1.42
CA/CL	3.43	1.85
I/D	5.62	2.35
S/FA	4.70	2.06
FA/S	4.13	1.83
CL/CA	2.01	1.42
D/I	-.37	.36
C/D	.07	1.14
NW/S	1.00	1.19
C/I	-.84	.97
TA/S	-.11	.77
S/I	-1.70	.33

Table 75: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6 Year: 1985

Variable	Kurtosis	Skewness
ROE	4.11	-1.93
SAL_GR	-.48	-.85
I/S	3.97	1.95
OPR_GR	7.25	2.45
STK_RT	.26	-.28
I/C	-1.42	.83
S/TA	-1.08	.66
I/D	2.44	1.63
D/C	5.01	2.30
S/NW	10.22	3.12
CA/CL	6.90	2.50
S/FA	2.90	1.82
FA/S	-1.64	.28
C/D	1.29	.85
CL/CA	4.50	2.09
NW/S	4.29	1.98
D/I	-1.81	.06
TA/S	3.74	1.80
C/I	.88	1.08
S/I	-.84	.33

Table 76: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6 Year: 1986

Variable	Kurtosis	Skewness
I/S	1.62	1.48
I/C	8.06	2.75
I/D	3.07	1.61
SAL_GR	5.03	2.04
S/TA	1.17	1.11
STK_RT	1.34	.21
S/NW	-.11	.89
D/C	6.81	2.48
ROE	2.03	1.69
S/FA	7.48	2.62
FA/S	-.38	.68
CA/CL	11.08	3.27
C/D	3.15	1.64
OPR_GR	12.60	3.53
NW/S	3.78	2.05
D/I	2.07	1.64
TA/S	7.49	2.65
CL/CA	12.88	3.58
C/I	-.29	.12
S/I	.77	.95

Table 77: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6 Year: 1987

Variable	Kurtosis	Skewness
I/S	5.56	2.24
I/C	-.08	1.03
I/D	-.05	.56
S/TA	7.20	2.61
STK_RT	1.18	.53
SAL_GR	3.37	1.44
CA/CL	7.70	2.67
D/C	5.30	2.06
CL/CA	.93	.80
S/FA	8.93	2.86
FA/S	-.76	.70
C/D	12.81	3.57
NW/S	-.94	-.54
S/NW	12.98	3.60
ROE	10.80	3.25
TA/S	-.69	-.79
OPR_GR	3.45	2.01
D/I	10.03	3.13
C/I	10.70	3.25
S/I	10.93	3.30

Table 78: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6 Year: 1988

Variable	Kurtosis	Skewness
I/S	-.11	1.09
SAL_GR	.92	-.14
I/C	.56	1.08
I/D	6.93	2.52
S/TA	2.44	1.57
STK_RT	8.43	2.82
S/NW	5.32	2.25
OPR_GR	1.76	-.97
ROE	-.53	.53
S/FA	1.73	1.78
C/D	7.49	2.50
D/C	10.73	3.26
CA/CL	10.71	3.26
CL/CA	4.25	2.16
FA/S	4.81	2.13
NW/S	3.51	1.61
D/I	.81	.12
TA/S	5.45	2.09
C/I	-.80	.25
S/I	-1.41	.24

Table 79: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Industry: 6 Year: 1989

Variable	Kurtosis	Skewness
STK_RT	.55	.96
I/S	.41	1.10
SAL_GR	.10	-.24
OPR_GR	10.29	3.11
S/TA	2.14	.44
I/C	5.17	2.05
S/NW	-.36	.16
I/D	2.40	1.34
ROE	5.55	2.45
S/FA	1.92	1.88
C/D	-1.30	-.14
CL/CA	10.18	3.11
D/C	11.49	3.37
FA/S	2.94	1.47
NW/S	-.34	.86
TA/S	4.63	1.88
CA/CL	11.98	3.46
C/I	10.09	3.12
D/I	10.85	3.29
S/I	8.85	2.88

Table 80: The table shows the skewness and the kurtosis of the unconstrained ratios for the indicated industry and year.

Variable	Year	Log Y	Log X	Std Dev	Zeta	Skewness
I/CA	82	4.21	4.77	0.71	0.79	0.64
I/TA		4.21	5.38	0.8	1.46	1.66
NW/TA		5.18	5.38	0.21	0.95	-0.88
TD/TA		4.72	5.38	0.41	1.61	0.99
FA/TA		4.73	5.38	0.57	1.14	0.53
CA/TA		4.77	5.38	0.52	1.17	0.79
CL/TA		4.63	5.38	0.38	1.97	1.48
C/CL		3.87	4.63	0.53	1.43	1.06
C/TA		3.87	5.38	0.6	2.52	5
I/CA	83	4.2	4.78	0.71	0.82	0.61
I/TA		4.2	5.41	0.77	1.57	1.44
NW/TA		5.19	5.41	0.22	1.00	-0.56
TD/TA		4.77	5.41	0.45	1.42	1.15
FA/TA		4.73	5.41	0.6	1.13	0.58
CA/TA		4.78	5.41	0.58	1.09	0.48
CL/TA		4.65	5.41	0.44	1.73	1.9
C/CL		3.92	4.65	0.5	1.46	1.21
C/TA		3.92	5.41	0.59	2.53	4.55
I/CA	84	4.34	4.82	0.74	0.65	0.54
I/TA		4.34	5.41	0.79	1.35	1.7
NW/TA		5.19	5.41	0.21	1.05	-0.67
TD/TA		4.79	5.41	0.41	1.51	1.56
FA/TA		4.76	5.41	0.66	0.98	0.5
CA/TA		4.82	5.41	0.53	1.11	0.57
CL/TA		4.67	5.41	0.42	1.76	2.08
C/CL		3.95	4.67	0.5	1.44	1.12
C/TA		3.95	5.41	0.6	2.43	4.15
I/CA	85	4.37	4.84	0.68	0.69	0.67
I/TA		4.37	5.44	0.73	1.47	1.37
NW/TA		5.21	5.44	0.22	1.05	-0.69
TD/TA		4.81	5.44	0.44	1.43	1.77
FA/TA		4.84	5.44	0.64	0.94	0.55
CA/TA		4.84	5.44	0.54	1.11	0.63
CL/TA		4.7	5.44	0.45	1.64	2.49
C/CL		4.01	4.7	0.5	1.38	0.98
C/TA		4.01	5.44	0.58	2.47	4.1

Table 81: This table shows the zeta and the skewness of the ratios where zeta=(log Y- log X)/log of std dev of the ratio, Y/X.

Variable	Year	Log Y	Log X	Std Dev	Zeta	Skewness
I/CA	86	4.38	4.88	0.78	0.64	0.9
I/TA		4.38	5.52	0.91	1.25	1.99
NW/TA		5.33	5.52	0.17	1.12	-0.53
TD/TA		4.87	5.52	0.42	1.55	2
FA/TA		4.8	5.52	0.8	0.90	0.82
CA/TA		4.88	5.52	0.6	1.07	0.5
CL/TA		4.75	5.52	0.43	1.79	2.62
C/CL		4.14	4.75	0.42	1.45	0.91
C/TA		4.14	5.52	0.52	2.65	3.58
I/CA	87	4.33	5.1	0.88	0.88	1.22
I/TA		4.33	5.72	1.01	1.38	2.06
NW/TA		5.51	5.72	0.21	1.00	-0.74
TD/TA		5.08	5.72	0.4	1.60	0.86
FA/TA		4.77	5.72	0.92	1.03	1.05
CA/TA		5.1	5.72	0.51	1.22	0.49
CL/TA		4.94	5.72	0.39	2.00	1.69
C/CL		4.3	4.94	0.43	1.49	0.97
C/TA		4.3	5.72	0.53	2.68	4.04
I/CA	88	4.43	5.21	0.93	0.84	1.05
I/TA		4.43	5.94	1.09	1.39	2.97
NW/TA		5.77	5.94	0.16	1.06	-0.7
TD/TA		5.26	5.94	0.37	1.84	0.76
FA/TA		4.62	5.94	1	1.32	1.68
CA/TA		5.21	5.94	0.58	1.26	0.89
CL/TA		5.13	5.94	0.37	2.19	0.94
C/CL		4.47	5.13	0.43	1.53	1.06
C/TA		4.47	5.94	0.57	2.58	2.23
I/CA	89	4.59	5.35	0.7	1.09	1.02
I/TA		4.59	6.04	0.93	1.56	2.3
NW/TA		5.87	6.04	0.15	1.13	-0.74
TD/TA		5.36	6.04	0.41	1.66	0.75
FA/TA		4.58	6.04	1.09	1.34	1.87
CA/TA		5.35	6.04	0.48	1.44	0.88
CL/TA		5.22	6.04	0.41	2.00	0.66
C/CL		4.54	5.22	0.45	1.51	0.79
C/TA		4.54	6.04	0.59	2.54	1.93
Correlation: 0.74						

Table 82: This table shows the zeta and the skewness of the ratios where zeta=(log Y- log X)/log of std dev of the ratio, Y/X.

Log of Sales

Firm Code	Adjusted R Sq.	Slope	Constant	Durbin Watson	Meaning
2075	0.65	0.10	-195.12	2.22	negative autocorrelation
2096	0.58	0.07	-137.55	1.55	positive autocorrelation
2111	-0.06	-0.02	51.78	0.85	positive autocorrelation
2118	-0.02	0.03	-50.60	0.75	positive autocorrelation
2119	0.39	0.05	-97.19	1.88	positive autocorrelation
2120	0.02	0.02	-25.77	1.55	positive autocorrelation
2125	0.29	0.03	-60.76	1.80	positive autocorrelation
2133	0.85	0.06	-114.35	1.57	positive autocorrelation
2137	0.29	0.06	-107.76	1.56	positive autocorrelation
2139	0.60	0.07	-132.71	2.20	negative autocorrelation
2158	0.13	0.08	-156.24	1.20	positive autocorrelation
2160	0.56	0.16	-308.37	1.51	positive autocorrelation
2164	0.95	0.08	-146.51	2.30	negative autocorrelation
2173					
2177	0.33	0.07	-134.52	1.05	positive autocorrelation
2184	0.77	-0.12	245.21	1.46	positive autocorrelation
2200	0.65	0.13	-247.32	2.05	negative autocorrelation
2203					
2208					
2210	0.01	0.05	-85.82	1.06	positive autocorrelation
2211					
2217					
2220	0.55	0.16	-303.48	1.16	positive autocorrelation
2224					
2225	0.52	0.03	-57.93	1.69	positive autocorrelation
2227	0.66	0.15	-296.74	2.35	negative autocorrelation
2231					
2235	0.15	0.14	-270.85	0.54	positive autocorrelation
2249	0.08	0.07	-134.26	1.24	positive autocorrelation
2254	0.59	0.36	-706.18	0.82	positive autocorrelation
2259					
2283					
2046	0.94	0.06	-122.81	0.97	positive autocorrelation
2061	0.96	0.07	-126.64	1.14	positive autocorrelation
2082					
2098	0.48	0.02	-26.10	2.37	negative autocorrelation
2116	0.79	0.03	-55.10	0.57	positive autocorrelation
2138	0.82	0.05	-89.70	1.17	positive autocorrelation
2141	0.67	0.05	-96.15	1.03	positive autocorrelation
2161	0.71	0.09	-171.51	0.95	positive autocorrelation
2195	0.88	0.08	-152.16	0.77	positive autocorrelation
2196	0.30	0.07	-130.85	1.78	positive autocorrelation
2226	0.56	0.02	-43.65	2.39	negative autocorrelation
2246	0.82	0.28	-557.31	2.10	negative autocorrelation
2094					
2140	0.90	0.13	-260.80	1.02	positive autocorrelation

Table 83: For each firm, this table shows the result of the regression and the meaning of Durbin Watson for log of Sales.

Log of Total Assets

Firm Code	Adjusted R Sq.	Slope	Constant	Durbin Watson	Meaning
2075	0.91	0.16	-303.84	0.95	positive autocorrelation
2096	0.83	0.07	-131.02	1.14	positive autocorrelation
2111	-0.04	0.02	-36.80	0.58	positive autocorrelation
2118	0.50	0.04	-65.51	2.93	negative autocorrelation
2119	0.77	0.10	-191.48	0.49	positive autocorrelation
2120	0.91	0.07	-124.79	2.06	negative autocorrelation
2125	0.79	0.09	-166.74	0.55	positive autocorrelation
2133	0.88	0.09	-175.47	0.54	positive autocorrelation
2137	0.27	0.03	-47.28	0.49	positive autocorrelation
2139	0.76	0.11	-214.46	0.72	positive autocorrelation
2158	0.39	0.06	-105.94	0.35	positive autocorrelation
2160	0.71	0.13	-257.80	0.45	positive autocorrelation
2164	0.87	0.11	-209.13	0.72	positive autocorrelation
2173	0.43	0.06	-117.89	0.89	positive autocorrelation
2177	0.31	0.03	-49.39	0.45	positive autocorrelation
2184	0.80	-0.08	168.31	1.04	positive autocorrelation
2200	0.69	0.11	-215.43	0.96	positive autocorrelation
2203	0.88	0.15	-301.68	1.20	positive autocorrelation
2208	0.77	0.12	-240.73	0.44	positive autocorrelation
2210	0.83	0.09	-168.93	0.65	positive autocorrelation
2211	0.73	0.11	-204.25	0.87	positive autocorrelation
2217	-0.07	-0.02	53.30	1.46	positive autocorrelation
2220	0.67	0.14	-271.78	1.59	positive autocorrelation
2224	0.81	0.06	-104.59	0.86	positive autocorrelation
2225	0.69	0.05	-97.92	1.01	positive autocorrelation
2227	0.75	0.14	-274.16	1.63	positive autocorrelation
2231	0.61	0.21	-406.39	0.93	positive autocorrelation
2235	0.63	0.17	-340.72	0.38	positive autocorrelation
2249	0.88	0.06	-118.53	1.36	positive autocorrelation
2254	0.66	0.33	-657.40	0.70	positive autocorrelation
2259	0.72	0.07	-135.35	0.69	positive autocorrelation
2283	0.59	0.13	-254.51	0.85	positive autocorrelation
2046	0.90	0.09	-168.42	1.05	positive autocorrelation
2061	0.96	0.11	-206.78	1.48	positive autocorrelation
2082	0.41	0.05	-99.71	0.60	positive autocorrelation
2098	0.78	0.02	-35.72	0.71	positive autocorrelation
2116	0.77	0.06	-114.12	0.74	positive autocorrelation
2138	0.77	0.10	-197.82	0.79	positive autocorrelation
2141	0.16	0.02	-38.87	0.52	positive autocorrelation
2161	0.78	0.08	-159.27	0.41	positive autocorrelation
2195	0.96	0.09	-164.75	1.23	positive autocorrelation
2196	0.17	0.01	-14.15	1.68	positive autocorrelation
2226	-0.09	0.00	3.35	1.81	positive autocorrelation
2246	0.90	0.17	-329.48	1.23	positive autocorrelation
2094	0.51	0.06	-112.25	1.66	positive autocorrelation
2140	0.82	0.16	-320.04	0.77	positive autocorrelation

Table 84: For each firm, this table shows the result of the regression and the meaning of Durbin Watson for log of Total Assets.

Log of Current Assets

Firm Code	Adjusted R Sq.	Slope	Constant	Durbin Watson	Meaning
2075	-0.10	0.00	-4.10	2.13	negative autocorrelation
2096	0.50	0.05	-85.98	1.22	positive autocorrelation
2111	-0.05	0.02	-43.84	0.78	positive autocorrelation
2118	-0.09	0.00	-3.69	1.56	positive autocorrelation
2119	0.59	0.06	-120.33	1.00	positive autocorrelation
2120	0.84	0.05	-90.46	2.18	negative autocorrelation
2125	0.15	0.06	-113.56	1.36	positive autocorrelation
2133	0.92	0.09	-174.59	1.07	positive autocorrelation
2137	0.79	0.07	-126.93	2.25	negative autocorrelation
2139	0.67	0.11	-209.95	0.89	positive autocorrelation
2158	0.19	0.04	-69.07	1.56	positive autocorrelation
2160	0.56	0.13	-252.45	0.74	positive autocorrelation
2164	0.91	0.07	-131.35	2.87	negative autocorrelation
2173	0.04	0.08	-150.32	1.59	positive autocorrelation
2177	0.98	0.12	-225.58	1.41	positive autocorrelation
2184	0.72	-0.09	179.19	0.98	positive autocorrelation
2200	0.60	0.07	-135.55	1.44	positive autocorrelation
2203	0.93	0.12	-242.24	1.89	positive autocorrelation
2208	0.83	0.13	-248.71	1.19	positive autocorrelation
2210	0.65	0.08	-161.42	1.40	positive autocorrelation
2211	0.62	0.11	-207.43	0.71	positive autocorrelation
2217	-0.10	-0.01	21.90	1.00	positive autocorrelation
2220	0.19	0.08	-146.23	1.38	positive autocorrelation
2224	0.86	0.08	-145.42	1.01	positive autocorrelation
2225	0.23	0.06	-113.89	0.48	positive autocorrelation
2227	0.69	0.17	-333.86	1.24	positive autocorrelation
2231	0.57	0.16	-308.89	1.18	positive autocorrelation
2235	0.62	0.17	-327.10	0.50	positive autocorrelation
2249	0.55	0.10	-196.20	1.47	positive autocorrelation
2254	0.59	0.19	-367.03	1.79	positive autocorrelation
2259	0.51	-0.07	136.55	1.32	positive autocorrelation
2283	0.24	-0.05	108.83	2.06	negative autocorrelation
2046	0.80	0.09	-165.88	0.68	positive autocorrelation
2061	0.80	0.06	-108.94	1.64	positive autocorrelation
2082	0.33	0.20	-390.72	1.33	positive autocorrelation
2098	0.82	0.02	-41.92	1.46	positive autocorrelation
2116	0.55	0.03	-51.04	1.61	positive autocorrelation
2138	0.66	0.09	-172.93	2.59	negative autocorrelation
2141	-0.08	0.01	-4.84	0.63	positive autocorrelation
2161	0.46	0.15	-294.29	0.99	positive autocorrelation
2195	0.03	0.02	-43.81	1.22	positive autocorrelation
2196	-0.02	0.02	-32.13	1.21	positive autocorrelation
2226	0.35	0.02	-43.76	1.22	positive autocorrelation
2246	0.94	0.20	-399.03	1.27	positive autocorrelation
2094	0.38	0.07	-129.89	2.00	no correlation
2140	0.58	0.21	-420.05	1.13	positive autocorrelation

Table 85: For each firm, this table shows the result of the regression and the meaning of Durbin Watson for log of Current Assets.

Log of Current Liabilities

Firm Code	Adjusted R Sq.	Slope	Constant	Durbin Watson	Meaning
2075	0.74	0.16	-305.04	2.07	negative autocorrelation
2096	0.75	0.13	-259.74	2.52	negative autocorrelation
2111	0.22	-0.05	102.96	1.28	positive autocorrelation
2118	-0.08	0.02	-33.86	1.45	positive autocorrelation
2119	0.88	0.04	-73.46	1.61	positive autocorrelation
2120	0.77	0.12	-226.21	2.62	negative autocorrelation
2125	0.58	0.04	-76.79	1.68	positive autocorrelation
2133	0.79	0.07	-139.65	0.67	positive autocorrelation
2137	0.05	-0.02	50.80	1.27	positive autocorrelation
2139	-0.03	0.03	-62.06	1.75	positive autocorrelation
2158	-0.07	0.02	-37.18	0.52	positive autocorrelation
2160	0.83	0.13	-260.01	1.50	positive autocorrelation
2164	0.86	0.08	-158.99	1.79	positive autocorrelation
2173	-0.03	0.03	-51.88	1.46	positive autocorrelation
2177	0.43	0.05	-88.55	0.96	positive autocorrelation
2184	0.65	-0.18	366.01	0.97	positive autocorrelation
2200	0.41	0.12	-224.15	1.36	positive autocorrelation
2203	0.60	0.15	-285.99	1.24	positive autocorrelation
2208	0.54	0.11	-224.55	0.78	positive autocorrelation
2210	0.43	0.10	-197.74	1.46	positive autocorrelation
2211	0.31	0.09	-168.56	1.42	positive autocorrelation
2217	0.43	0.11	-210.15	1.69	positive autocorrelation
2220	0.43	0.17	-326.39	1.14	positive autocorrelation
2224	0.58	-0.04	93.01	1.10	positive autocorrelation
2225	-0.04	0.01	-25.43	2.20	negative autocorrelation
2227	0.32	0.12	-242.24	1.33	positive autocorrelation
2231	0.53	0.27	-528.40	1.08	positive autocorrelation
2235	0.27	0.18	-353.17	0.88	positive autocorrelation
2249	0.16	0.05	-89.25	1.52	positive autocorrelation
2254	0.76	0.48	-940.75	1.31	positive autocorrelation
2259	0.02	0.02	-36.32	1.12	positive autocorrelation
2283	-0.02	0.06	-116.88	1.03	positive autocorrelation
2046	0.75	0.14	-273.95	1.11	positive autocorrelation
2061	0.87	0.08	-146.00	1.83	positive autocorrelation
2082	0.21	0.09	-164.94	0.75	positive autocorrelation
2098	0.51	0.03	-62.20	2.35	negative autocorrelation
2116	0.22	0.03	-60.21	1.50	positive autocorrelation
2138	0.73	0.13	-256.18	0.84	positive autocorrelation
2141	-0.03	-0.01	21.13	0.90	positive autocorrelation
2161	0.38	0.08	-158.18	0.67	positive autocorrelation
2195	0.78	0.09	-165.90	0.95	positive autocorrelation
2196	0.19	0.05	-90.48	1.53	positive autocorrelation
2226	0.10	-0.02	39.23	1.94	positive autocorrelation
2246	0.92	0.21	-409.69	2.10	negative autocorrelation
2094	0.58	0.05	-93.17	2.57	negative autocorrelation
2140	0.89	0.12	-233.24	1.37	positive autocorrelation

Table 86: For each firm, this table shows the result of the regression and the meaning of Durbin Watson for log of Current Liabilities.

Log of Operating Expenses

Firm Code	Adjusted R Sq.	Slope	Constant	Durbin Watson	Meaning
2075	0.03	0.04	-74.80	1.91	positive autocorrelation
2096					
2111	0.30	-0.10	208.29	1.87	positive autocorrelation
2118	-0.02	0.03	-54.90	0.76	positive autocorrelation
2119	0.21	0.03	-57.47	1.55	positive autocorrelation
2120					
2125	-0.08	-0.01	23.24	1.55	positive autocorrelation
2133	0.78	0.05	-95.39	2.33	negative autocorrelation
2137	0.44	0.07	-133.84	1.65	positive autocorrelation
2139	0.53	0.07	-136.80	1.64	positive autocorrelation
2158					
2160	0.76	0.19	-369.67	1.61	positive autocorrelation
2164	0.92	0.07	-140.43	2.27	negative autocorrelation
2173					
2177	0.18	0.05	-101.62	0.94	positive autocorrelation
2184	0.59	-0.10	195.86	0.88	positive autocorrelation
2200	0.67	0.16	-304.73	2.15	negative autocorrelation
2203					
2208					
2210	0.01	0.05	-97.24	1.09	positive autocorrelation
2211					
2217					
2220	0.33	0.16	-310.65	1.03	positive autocorrelation
2224					
2225	-0.07	0.01	-13.71	2.05	negative autocorrelation
2227	0.51	0.12	-241.28	1.89	positive autocorrelation
2231					
2235	0.08	0.12	-241.96	0.48	positive autocorrelation
2249	-0.10	0.01	-6.98	2.00	no correlation
2254					
2259					
2283					
2046	0.95	0.07	-132.13	1.00	positive autocorrelation
2061	0.96	0.07	-135.79	1.12	positive autocorrelation
2082					
2098	0.35	0.01	-19.32	2.16	negative autocorrelation
2116	0.81	0.03	-56.81	0.51	positive autocorrelation
2138	0.77	0.07	-131.35	0.75	positive autocorrelation
2141	0.75	0.05	-91.17	2.15	negative autocorrelation
2161	0.75	0.09	-180.03	1.23	positive autocorrelation
2195	0.89	0.11	-220.70	1.25	positive autocorrelation
2196	0.24	0.07	-141.32	2.29	negative autocorrelation
2226	0.76	0.03	-61.35	2.34	negative autocorrelation
2246	0.87	0.43	-846.27	0.92	positive autocorrelation
2094					
2140	0.88	0.18	-350.89	1.12	positive autocorrelation

Table 87: For each firm, this table shows the result of the regression and the meaning of Durbin Watson for log of Operating Expenses.

Runs Test

Firm Code	Change in Current Rat.	Change in Dividends	Change in Gross Margin	Change in Net Income	Change in Operat. Profits	Change in Mkt. Price	Change in Sales
2046	1.00	0.68	0.76	0.13	0.36	0.76	0.36
2061	0.36	0.40	0.36	1.00	0.76	0.36	0.01
2075	0.76	0.13	0.13	0.36	0.01	0.36	0.76
2082	0.76	0.40		0.76	0.13	0.36	
2094	0.76	0.76		0.76	0.76	0.36	
2096	0.36	1.00	0.76	1.00	0.36	1.00	0.76
2098	0.76	0.68	0.13	0.36	0.36	0.76	0.36
2111	0.13	1.00	0.03	1.00	0.36		1.00
2116	0.76	0.30	0.76	0.76	0.76	0.40	0.76
2118	0.13	0.36	0.36	1.00	1.00	0.76	1.00
2119	0.76	1.00	0.76	0.36	0.36		0.36
2120	0.13	0.30	0.76	0.36	0.36	1.00	1.00
2125	0.13	0.15	0.36	0.01	0.76	1.00	0.76
2133	0.13	1.00	0.76	0.36	0.76	1.00	1.00
2137	1.00	0.57	0.36	1.00	1.00	1.00	0.76
2138	0.76	0.84	0.76	0.76	1.00	0.13	0.13
2139	1.00	0.76	1.00	0.76	0.76	0.76	0.36
2140	0.76	0.03	0.13	1.00	1.00	1.00	0.76
2141	0.13	0.21	1.00	0.76	1.00	0.36	0.76
2158	0.76	0.76	1.00	0.36	0.36	0.76	0.36
2160	0.36	1.00	0.76	0.76	0.36		0.36
2161	0.36	0.76	0.36	1.00	0.36		0.36
2162	0.36	1.00	1.00	0.76	0.76	0.36	0.76
2164	0.36	0.15	0.76	0.76	0.13	0.36	0.13
2173	1.00	0.09		1.00	0.76		
2177	0.76	0.13	0.36	0.76	0.36	1.00	0.13
2184	0.76	0.09	0.36	1.00	0.13	0.40	0.13
2185	0.13	0.36	0.76	1.00	1.00	0.76	0.76
2195	1.00		0.13	0.76	1.00	1.00	0.76
2196	0.36	0.76	1.00	0.36	0.36	0.76	0.76
2200	0.76	1.00	0.36	0.36	0.36		0.76
2203	0.13	1.00		0.76	0.76		
2208	0.36	1.00		0.76	1.00		
2210	1.00	0.36	0.36	0.36	0.76		1.00
2211	1.00	0.76		0.76	0.76	0.76	
2217	1.00	1.00		0.36	0.76	1.00	
2220	0.76	0.13	0.76	0.76	0.76	1.00	0.76
2224	0.13	0.91		0.76	0.76	0.36	
2225	1.00	0.15	0.13	1.00	0.13		0.76
2226	0.36	0.76	0.76	0.76	0.76	0.76	0.36
2227	0.13	0.03	0.13	0.76	0.13		0.76
2231	0.36	1.00		0.76	0.76		
2234	0.76	0.32	0.76	1.00	1.00	0.36	1.00
2235	1.00	0.37	0.76	0.76	0.13		0.76
2246	0.36	0.76	0.36	0.36	0.76	0.13	0.13
2249	0.13	0.91	0.76	0.36	0.36		0.36
2254	0.76		0.36	0.36	0.76	0.13	1.00
2259	0.13	1.00		0.76	0.76	0.36	
2276	1.00	0.76		0.13	0.76	0.76	
2283	0.76	0.21		1.00	1.00	1.00	

Table 88: For each firm, this table shows the P-values of the runs test for the changes of Current Ratio, Dividends, Gross Margin, Net Income, operating Profit, Market Prices and Sales.

Runs Test

Firm Code	Residual of Current Assets	Residual of Current Lia.	Residual of Operat. Expenses	Residual of Sales	Residual of Total Assets
2075	1.00	0.76	0.40	0.40	0.15
2096	0.36	1.00		1.00	0.05
2111	0.03	0.21	0.40	0.04	0.04
2118	0.40	0.04	0.57	0.57	1.00
2119	0.04	0.40	0.40	1.00	0.04
2120	0.76	1.00		0.84	0.91
2125	0.36	0.36	0.76	0.40	0.04
2133	0.40	0.03	0.91	1.00	0.04
2137	0.84	0.36	1.00	0.84	0.04
2139	0.05	0.84	0.40	1.00	0.05
2158	0.40	0.05		0.13	0.04
2160	0.36	0.40	0.40	0.57	0.03
2164	0.30	0.68	0.76	0.76	0.04
2173	0.21	0.40			0.36
2177	1.00	0.36	0.15	0.36	0.03
2184	0.36	0.13	0.76	1.00	0.21
2200	1.00	0.76	0.84	0.76	0.04
2203	0.40	0.21			0.84
2208	0.40	0.05			0.04
2210	0.57	1.00	0.05	0.05	0.36
2211	0.03	0.40			0.04
2217	0.04	0.40			0.84
2220	0.15	0.36	0.15	0.21	0.36
2224	0.03	0.57			0.03
2225	0.40	0.76	0.76	0.84	0.13
2227	0.03	0.15	1.00	1.00	0.13
2231	0.15	0.09			0.15
2235	0.04	0.36	0.04	0.04	0.03
2249	0.13	0.91	0.91	0.40	0.03
2254	1.00	0.84		0.05	0.15
2259	0.76	0.15			0.05
2283	0.57	0.32			0.05
2046	0.40	0.15	0.15	0.15	0.15
2061	0.76	0.76	0.15	0.21	0.40
2082	0.15	0.40			0.05
2098	0.84	0.68	0.91	0.91	0.04
2116	1.00	1.00	0.04	0.04	0.04
2138	1.00	0.03	0.03	0.15	0.04
2141	0.09	0.04	1.00	0.36	0.04
2161	0.05	0.03	0.84	0.36	0.04
2195	0.04	0.15	0.57	0.03	0.13
2196	0.13	0.76	0.40	0.21	0.40
2226	0.21	1.00	0.76	1.00	1.00
2246	0.40	0.68	0.13	0.84	0.15
2094	0.36	0.36			1.00
2140	0.40	0.05	0.40	0.21	0.15

Table 89: For each firm, this table shows the P-values of the runs test for the residuals of Current Assets, Current Liabilities, operating Profit, Sales and Total Assets.

Serial Correlation

Firm code	Sales	Operating Profit	Net Income	Dividends	Market Price
2075	0.92	0.98	0.78	-0.04	0.31
2096	0.43	-0.01	0.33	0.23	0.50
2111	0.13	0.52	0.49	0.32	
2118	0.53	-0.09	-0.26	0.16	0.79
2119	0.43	0.42	0.53	0.18	
2120	-0.02	-0.07	0.30	0.11	0.13
2125	0.50	0.94	0.90	0.32	0.40
2133	0.91	0.92	0.97	0.47	0.50
2137	0.25	0.36	0.55	0.61	0.52
2139	0.52	0.33	-0.10	0.06	0.60
2158	0.09	0.14	0.31	0.58	0.34
2160	0.89	0.66	0.58	-0.25	
2164	0.97	0.91	0.76	0.94	0.55
2173		0.58	0.54	0.64	
2177	0.54	0.41	0.24	0.67	0.62
2184	0.78	0.66	0.61	0.65	0.79
2200	0.79	0.05	0.02	0.21	
2203		0.44	0.31	0.28	
2208		-0.22	0.63	0.15	
2210	0.40	0.07	0.40	-0.02	
2211		0.60	0.38	0.62	0.81
2217		-0.26	-0.08	-0.09	0.36
2220	0.80	0.18	0.31	0.43	0.47
2224		0.12	0.24	0.65	0.52
2225	0.51	0.75	0.68	0.14	
2227	0.30	0.62	0.74	-0.00	
2231		0.85	0.56	0.58	
2235	0.56	0.81	0.57	0.29	
2249	0.09	0.02	0.05	0.05	
2254	0.86	-0.94	0.87	0.97	0.00
2259		0.79	0.43	-0.04	0.89
2283		0.90	0.87	0.54	0.31
2046	0.97	0.81	0.32	0.37	0.17
2061	0.98	0.79	0.46	0.20	0.17
2082		0.70	0.85	0.64	0.66
2098	0.44	0.33	-0.52	-0.04	0.93
2116	0.95	0.58	-0.34	0.34	0.69
2138	0.83	0.84	0.43	0.43	0.60
2141	0.90	0.34	0.28	0.31	0.36
2161	0.89	0.48	0.56	0.36	
2195	0.95	0.38	0.17	0.87	0.59
2196	0.16	0.45	0.33	0.59	0.71
2226	0.57	0.16	0.34	0.30	0.69
2246	0.96	0.86	0.84	0.27	0.70
2094		0.42	-0.20	0.37	0.58
2140	0.94	0.92	0.01	0.67	0.45

Table 90: For each firm, this table shows the P-values of the serial correlation test for Sales, operating Profit, Net Income, Dividends and Market Price.

Runs test: Comparing the randomness
of residuals and differences

Firm Code	Residual of Sales	Difference in Sales
2046	0.15	0.36
2061	0.21	0.01
2075	0.40	0.76
2096	1.00	0.76
2098	0.91	0.36
2111	0.04	1.00
2116	0.04	0.76
2118	0.57	1.00
2119	1.00	0.36
2120	0.84	1.00
2125	0.40	0.76
2133	1.00	1.00
2137	0.84	0.76
2138	0.15	0.13
2139	1.00	0.36
2140	0.21	0.76
2141	0.36	0.76
2158	0.13	0.36
2160	0.57	0.36
2161	0.36	0.36
2164	0.76	0.13
2177	0.36	0.13
2184	1.00	0.13
2195	0.03	0.76
2196	0.21	0.76
2200	0.76	0.76
2210	0.05	1.00
2220	0.21	0.76
2225	0.84	0.76
2226	1.00	0.36
2227	1.00	0.76
2235	0.04	0.76
2246	0.84	0.13
2249	0.40	0.36
2254	0.05	1.00
19 cases in 35 yield Runs Tests for Changes with P-values higher than those for Residuals.		

Table 91: This table compares results of the Runs test for the change in Sales and Residuals of Sales.