Incorporating complementary ratios in the analysis of financial statements [1]

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ABSTRACT: Ratios are commonly used to extract information from accounting reports. However, ratios present only part of the information available in their two components. This article infers the functional form of the information discarded by ratios. It then develops ratio complements that incorporate the discarded information, showing examples of their use and discussing the benefits obtained. Ratio complements can detect size-related anomalies of the firm that standard ratios do not recognise, providing a measure of size efficiency with which other financial features can be compared. Consideration of both the ratio and its complement leads to the development of graphical representations of financial features. As well as being intuitive, these graphical representations are a first step towards the automation of financial diagnostics and may lead to a more technology-oriented analysis of financial statements.

KEYWORDS: Financial Analysis, Financial Ratios, Complementary Ratios, Firm Size, Scale Effect, Self-Organizing Maps.

JEL CODES: C44, C45, G30, L25, M41.

Introduction

Accounting reports are an important source of information for managers, investors, and financial analysts. Ratios are the usual instruments for extracting this information. They are

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[1] This is an updated and improved version of an article published in 1994. References to contents here should be made as follows: Trigueiros, D., (1994), Incorporating complementary ratios in the analysis of financial statements, *Accounting, Management, and Information technologies* (Elsevier), Vol. 4, no. 3, pp. 149-162.

supposed to remove the effect of size from accounting variables and to highlight noteworthy features of the firm such as profitability or liquidity. Foster (1986) offers a detailed description and thorough discussion of the use of ratios in the analysis of financial statements.

Ratios present only part of the information available in their two components. In the first instance, ratios discard information regarding the size of the firm being analysed, thus allowing the comparison with firms of different sizes. However, size is not the only piece of information discarded by ratios. Ratios also discard information from which the effect of size has been removed. This article infers the functional form of the *size-free* information discarded by ratios. It then discusses its usefulness, showing that such information, being complementary to that provided by ratios, is valuable for financial analysis.

Complements of ratios can detect size-related anomalies that standard ratios do not recognise, providing a measure of efficiency regarding size, against which other financial features of the firm such as profitability or leverage can be compared. By highlighting deviations from what is expected for a firm of that size, the complement of a ratio will convey useful information about a firm's competitive advantage or disadvantage.

The consideration of both the ratio and its complement leads to the development of graphical representations of financial features. The final section of the article uses a particular kind of Neural Network, the Self-Organized Map (Kohonen, 1986) to discretize the developed graphical representations as a first step towards automating the financial diagnosis of firms.

The concepts developed here allow a thorough exploring of the possibilities offered by existing technologies, leading to more accurate and intuitive analysis.

The Complement of a Ratio

This section shows that, given two items, y and x, used as the numerator and the denominator of a ratio, the whole size-free information contained in them can be expressed in terms of the ratio itself, y/x, plus a complement. We also point out that such a complement is likely to be valuable for financial analysis.

The Size-Free Information Contained in Two items

Numerical items found in databases of accounting reports of firms can be viewed as statistical variables where each report is a case. For a given reported item, say Fixed Assets or Sales, a cross-section sample can be drawn from reports belonging to the same period.

Studies on the statistical distributions of cross-sections of accounting items have uncovered two facts. First, the probability density function governing the occurrence of most items tends to be lognormal (McLeay, 1986; Trigueiros, 1995; Falta and Willett, 2011).[2] Second, due to the common effect of size, items belonging to the same report share a great deal of their variability (McLeay and Trigueiros, 2002).[3] Therefore, the variability of the logarithm of items taken from the same report should be explained as the common effect of size, σ, plus size-free variability ε_i, particular to each item. That is, item i is explained as

$$\log x_i = \mu_i + \sigma + \varepsilon_i \qquad (1)$$

where the μ_i are expected values of $\log x_i$ for item i. After removing the effect of size, the variability remaining in item i is ε_i. Notice that ε_i is the logarithm of a ratio in which the denominator is the statistical effect of size, and the numerator is the deviation of item i from

[2] It has long been established that income, wealth, firm size, and other economic accruals resulting from the accumulation of random amounts, obey a multiplicative law of probabilities such as the Pareto or the Gibrat laws, and not an additive law such as the Central Limit Theorem (Ijiri and Simon, 1964; Singh and Whittington, 1968). As the statistical behaviour of financial statement numbers approximates other economic accruals, their cross-section distributions obey a multiplicative law of probabilities (McLeay, 1986; Tippett, 1990) and ratios formed from them are multiplicative as well (Johnson *et al.*, 1994).

[3] Not only is it an empirical fact that the financial statements of large firms contain reported numbers that are many orders of magnitude larger than those in the accounts of small firms, but there are also compelling economic reasons to support the conviction that each firm's actual size greatly influences the overall magnitude of numbers reported in its accounts. Indeed, if items such as Sales or Earnings were not closely related to size, then profitability and dividend yield would be diluted by any increase in size and firms would avoid growing.

expected. Such ratio reflects the proportion to which item i differs from that expected in firms of that size. That is, the ε_i are size-free Sales, Working Capital, Fixed Assets, and so on. An ε_i larger than zero denotes an item i larger than expected for that industry, irrespective of size.

Assessing the Statistical Effect of Size

The ε_i would be useful for the analysis of financial statements as they convey size-free information in the same way ratios do. To isolate every ε_i, it is first necessary to estimate the effect of size, the σ, and then subtract it from the logarithm of each item. Since, μ_i is a "fixed" effect and σ arguably is a "random" effect, it follows that (1) is a variance components model. The mixed effects estimation of μ_i and σ is time-consuming as it calls for the sampling, inside each report, among the usable items.[4]

Instead of such arduous σ estimate, some proxy for size such as Total Assets or Sales (Dang *et al.*, 2018) can be used. However, the σ estimated in this way contain spurious variability, the one specific to the proxy, and may also be biased. A better, yet not perfect alternative consists of building, inside each report, averages of several log-transformed, mean adjusted items. In this way, the variability particular to each item, ε_i, is smoothed away so that only the common one remains. In fact, by applying (1) to M items x_i with $i = 1, M$ from the same report, that is,

$$
\begin{gathered}
\log x_1 - \mu_1 = \sigma + \varepsilon_1 \\
\log x_2 - \mu_2 = \sigma + \varepsilon_2 \\
\vdots \\
\log x_M - \mu_M = \sigma + \varepsilon_M
\end{gathered}
$$

and averaging, we obtain:

[4] Usable items: those where the effect of size is present, the sources of variability are multiplicative, the level of aggregation is not high (Total Assets would not be adequate), have small size-free correlation with other items and small number of missing cases. Negative cases are not a problem because they too reflect size (Trigueiros, 2019). To reduce standard errors, as many items as possible should be used.

$$\sigma = \frac{1}{M}\sum_{i=1}^{M}(\log x_i - \mu_i) - \frac{1}{M}(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_M)$$

Since an average of independent random deviates tends to zero with $1/M$,

$$\sigma \approx \frac{1}{M}\sum_{i=1}^{M}(\log x_i - \mu_i) \qquad (2)$$

for large enough M. Items adequate for estimating σ using formula (2) should have small variability of their own and their ε_i should not be correlated, otherwise, a larger M may be required. It is easy to verify that μ_i and σ estimated in this expedite way are, in the most common situations, comparable to the estimates provided by mixed models. Henceforward, μ_i and σ refer to the respective parameters, estimated as highlighted in (2).

Both ε_i and σ are logarithms of proportions, being independent of scale and unit of measure. The log-size, σ, estimates, on a logarithmic scale, the proportion to which the size of a given firm is larger or smaller than the industry average. Therefore, (2) can be interpreted as a means of performing the task opposite to ratios. Ratios remove size thus revealing deviations from size while (2) removes deviations from size thus revealing size.

Assessing the Complementary Information Discarded by Ratios

Since any pair of items, $\{x, y\}$, conveys two-dimensional information and ratios are just one variable, when ratios are used instead of their components some information is discarded. Not only size information, but also size-independent information that might be of interest for financial analysis is discarded. Given (1), the information conveyed by the ratio y/x can be written on a logarithmic scale as the subtraction of two deviations from expected in firms of that size, that is,

$$\text{since} \quad \begin{matrix} \varepsilon_y = \log y - \mu_y - \sigma \\ \varepsilon_x = \log x - \mu_x - \sigma \end{matrix} \quad \text{then} \quad \varepsilon_y - \varepsilon_x = \log\frac{y}{x} - \mu_{y/x}, \qquad (3)$$

Let us define two Cartesian coordinates in which the ε_y are measured along the Y-axis and the ε_x along the X-axis. All the size-free information conveyed by items y and x about one firm, will be represented by *locus* $\{\varepsilon_y, \varepsilon_x\}$ in this coordinate system. Now we rotate this system 45^o anti-clockwise: we apply the transformation H to each pair $\{\varepsilon_x, \varepsilon_y\}$ or to a matrix D containing a cross-section of many such pairs in rows:

$$D^r = DH \quad with \quad H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad (4)$$

We thus obtain new coordinates in which the X-axis measures $\varepsilon_y - \varepsilon_x$ and the Y-axis measures $\varepsilon_y + \varepsilon_x$. As seen in (3), $\varepsilon_y - \varepsilon_x$ is, on a logarithmic scale, the information conveyed by the ratio y/x. Since the new Y-axis is orthogonal to the axis assessing the information conveyed by the ratio, we can be sure that all the information not accounted for by the ratio will be contained in $\varepsilon_y + \varepsilon_x$. That is, of all the size-free information contained in items x and y, $\varepsilon_y + \varepsilon_x$ conveys the one discarded by the ratio y/x.

As can be deduced from (3), the information in $\varepsilon_y + \varepsilon_x$ is assessed on an ordinary (not logarithmic) scale by the new ratio xy/s^2. We call this new ratio the "complement" of y/x. Its denominator, s^2, is the squared effect of size ($\sigma = \log s$) on an ordinary scale, estimated as suggested in (2) or in a more adequate way.

The usefulness of the complementary ratio

Ratios y/x and xy/s^2 measure the two orthogonal dimensions of an observation which, in turn, is associated with a financial feature (liquidity, profitability and so on), but it remains to be seen whether complements of ratios contain useful information beyond that of ratios. To begin with, if ε_y is strongly correlated to ε_x, this decomposition of information becomes less attractive since correlation means redundancy. An instance where ratio complements are not very useful for financial analysis is when the denominator of the considered ratio is a size proxy. Indeed, size removal by ratios is accomplished in two ways (Lev and Sunder, 1979):

- **Explicitly**, when the denominator of the ratio is selected to reflect size (Total Assets or Sales are typical choices). Ratios explicitly removing size are meant to measure whether a particular item is large or small when compared with the size of the firm.
- **Implicitly**, when the denominator of the ratio is selected to produce a desired contrast with the numerator. Ratios implicitly removing size are meant to measure a financial feature, that is, whether a particular item is large or small when compared with other item, irrespective of size. Size, however, is removed all the same.

For example, in the two ratios Working Capital to Total Assets and Current Assets to Current Liabilities, the former assesses liquidity by comparing Working Capital with a proxy for size (Total Assets), while the latter compares short-term assets with short-term liabilities regardless of size. Ratios meant to explicitly remove size will benefit little from the pairing with their complements because their denominators have little else to offer besides size.[5]

The usefulness of the complementary ratio rests on the ability to identify deviations from expected for firms of that size. For example, it may happen that, in a firm, liquidity agrees with the Current Ratio norm, but both Current Assets and Current Liabilities are larger or smaller than expected for firms of that size. The complementary ratio will point out such deviation in a more precise and comparable way than that offered by the Working Capital to Total Assets ratio, which is just an *ad hoc* substitute, not the true complement of the Current Ratio. Later in the article other examples will be offered of the usefulness ratio complements.

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[5] When applied to ratios, terms such as "complement" and "pair" have an analytical meaning that is distinct from the meaning given here. For example, Interest Cover is used in conjunction with Financial Structure ratios. These two pieces of information are complementary for reasons of financial analysis, not because they are orthogonal.

Two-Dimensional Representations of Financial Features

Graphical, two-dimensional information is more intuitive than the separate examination of two dimensions and trajectories are more accurate and easier to interpret than simple time-histories. Moreover, two-dimensional (bivariate) distributions often are difficult to describe functionally. Therefore, instead of analysing complementary ratios separately, practitioners should privilege the joint analysis of ratios and their complements provided by graphical tools.

The Rotated Residual Plot

The Rotated Residual Plot (RRP) is a scatterplot in which the X-axis measures, on a logarithmic scale, deviations of y/x from the industry average. For conveniently selected y and x, this axis is supposed to capture a financial feature of the firm such as profitability. The Y-axis measures deviations of xy/s^2 from the industry average, that is, the joint deviation of y and x from the expected in that industry in firms of that size.

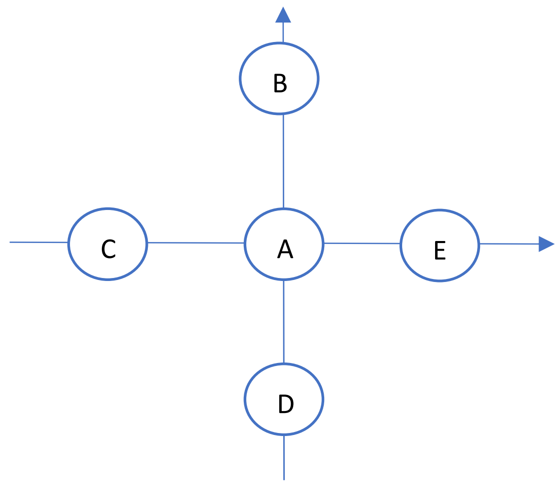
The RRP is a 45^o anti-clockwise rotation of a scatterplot of ε_y with ε_x. As seen in (4), this rotation leads to a new X-axis assessing $\varepsilon_y - \varepsilon_x$ and a new Y-axis assessing $\varepsilon_y + \varepsilon_x$.

Cross-sectional ratio analysis is based on the magnitude of deviations from industry norms. Norms are important only in that they are needed for calculating such deviations. The RRP plots deviations from averages in log space, that is, median values.

When showing several years in the same RRP, each year can be mean adjusted separately, thus accounting for trends that affect one whole industry. If not accounted for, these trends would introduce fluctuations in ratio norms, making it misleading to compare ratios in different years.

Recall that ratios are supposed to capture financial features, and, in this sense, the diagnostics provided by the RRP can be described in terms of the respective positions as:

Figure 1. Financial analysis based on the location of firms in the RRP. For a given ratio, say, the Current Ratio, X- and Y-axes measure $\varepsilon_y - \varepsilon_x$ and $\varepsilon_y + \varepsilon_x$ respectively. Each *locus* in the RRP is an industry-adjusted two-dimensional representation of the ratio's feature.



1. **Position is near "A"** both the feature and its magnitude when compared with the size of the firm are near that expected for the industry. For example, in the case of liquidity, this position means that the proportion of Current Assets to Current Liabilities is near the average and these items are also in the usual proportion to the size of the firm.
2. **Position is along the "B" line** the feature is near the expected, but its magnitude is larger than expected in firms of that size. For example, the proportion of Current Assets to Current Liabilities is near average, but these items are larger than expected in firms of that size.
3. **Position is along "C"** denote a magnitude of the feature near expected given the size of the firm but the feature itself is below expected. Continuing with liquidity, this position corresponds to Current Assets falling short of the expected proportion to Current Liabilities but these items being in balance with size.
4. **Position is along "D"** show that the feature is near expected but its magnitude, given the size of the firm, is small. For example, liquidity might agree with the norm but both

Current Assets and Current Liabilities are smaller than expected for firms of that size.

5. **Position is along "E"** show that the magnitude of items conforms with the industry average in the case of firms of that size, but their proportion is above expected. For example, there is an excess of short-term assets over short-term liabilities, even when the items reflecting liquidity are in balance with the size of the firm.

Figure 1 shows diagnostics to infer from locations in the RRP. Positions between axes induce a combination of two of the above diagnostics. For example, when the position of a firm is between "C" and "D", both the feature and its magnitude given the size of the firm are below expectation for that industry. This is frequent when assessing profitability and it means firms too big for the generated earnings.

How to Build the RRP

We extracted from the Micro-EXSTAT database (EXTEL LTD, UK; see Board *et al.*, 1991) all the reports of firms belonging to the Food Manufacturing industry in the UK during the period 1983-1987. Then two items were selected from these reports: EBIT (Earnings Before Interest and Tax) and NW (Net Worth, the book value of shareholders' equity). These items are the components of a profitability ratio measuring the percent operating profit generated by the firm's unit value, thus showing the operating efficiency of the firm's worth. These two item values were then returned to their original,[6] and the symmetric *ad hoc* transformation

$$x \mapsto \log x \quad \text{for } x > 1$$

$$x \mapsto -\log|x| \quad \text{for } x < -1$$

$$x \mapsto 0 \text{ for } x \approx [-1,1]$$

[6] Typically, numbers stored in databases of accounting reports are scaled down in relation to the true figures. In the COMPUSTAT file of US quoted companies, for example, accounting items are divided by one million before being stored. Therefore, numbers smaller than 1 abound. Before applying logarithmic transformations, numbers must be descaled since values smaller than 1 become ambiguous when transformed.

was applied to them (Snedecor and Cochran, 1965). Negative NW cases were discarded.[7] Transformed items were then mean adjusted separately by year, for the period 1983-1987.

Positive and negative EBIT were mean adjusted separately. This is common practice in ratio analysis, since profits and losses are two different populations and should not be mixed in the same sample. Accordingly, the negative- EBIT firms were placed in the third quadrant of the RRP, away from the positive-EBIT ones, as shown in figure 4.[8]

The estimated σ is obtained by averaging the logs of six items as in (2):

$$\sigma = \frac{1}{6}[\log \text{Sales} + \log \text{Wages} + \log \text{Number of Employees} + \log \text{Debtors} + \log \text{Current Liabilities} + \log \text{Current Assets}] \quad (5)$$

As said, log items in (5) were previously mean adjusted separately by year.

Next, for all the firms in the Food Manufacturing industry, we calculated the Y and X coordinates in the RRP, in the same way for positive- and negative-EBIT observations. If

$$\varepsilon_{ebit} = \log EBIT - \overline{\log EBIT} - \sigma \quad \text{and} \quad \varepsilon_{nw} = \log NW - \overline{\log NW} - \sigma,$$

where $\overline{\log \text{EBIT}}$ and $\overline{\log \text{NW}}$ are the industry averages, then the two axes of the RRP are

$$Y = \varepsilon_{ebit} + \varepsilon_{nw} \quad and \quad X = \varepsilon_{ebit} - \varepsilon_{nw}.$$

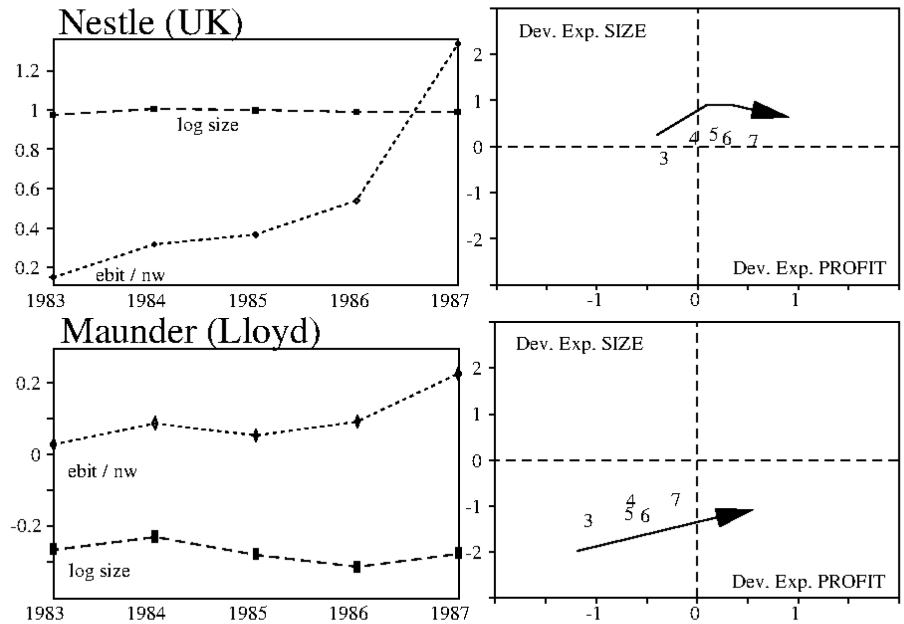
The RRP is the scatterplot showing, for the selected industry, the *loci* of firms according to the estimated X and Y.

How to Use the RRP

After building the food manufacturers' RRP, four firms were selected from this industry and their profitability was examined. Figures 2 and 3 display, on the left, time-histories of the ratio EBIT to NW and estimated σ; on the right, their trajectories in the RRP. The nearer a firm is to the centre of the RRP the less it diverges from the industry norm.

[7] A ratio is ambiguous and therefore useless when both the numerator and denominator can take negative values.
[8] Foster (1986) and Lev and Sunder (1979) offer a more detailed discussion of this practice.

Figure 2: On the left, the time-history of the ratio EBIT/NW and σ during five years for firms Nestle (UK) and Maunder (Lloyd). On the right, the respective trajectory drawn in the RRP during the same period. Marks 1 to 7 refer to 1983 to 1987.

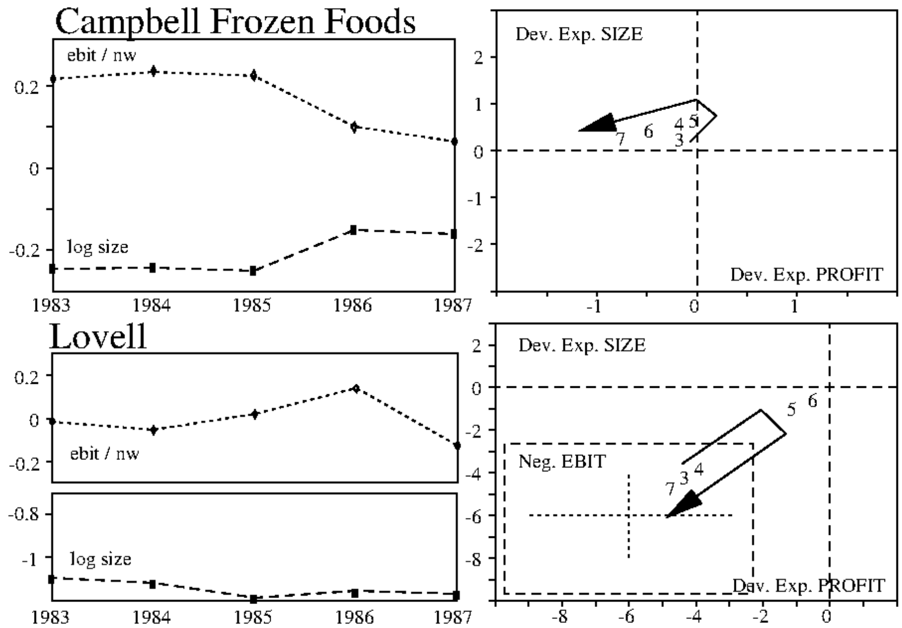


Recall that the X-axis of the RRP measures, on a logarithmic scale, percent deviations from the average operating profitability in the Food Manufacturing industry. The Y-axis measures, on the same scale, how Earnings and Net Worth, jointly considered, diverge from expected in firms of that size.

The first quadrant of the RRP contains firms with both Earnings and Net Worth above expected given the size of the firm. The second quadrant means Net Worth above expected but Earnings below expected, and so on. The desirable position in the RRP is (in this case) along the positive X-axis, as far away as possible from the origin since distance to the origin depicts advantageous operating profitability against the industry.

Trajectories parallel to the X-axis depict firms increasing or decreasing in efficiency when compared to the industry. Trajectories parallel to the Y-axis depict firms expanding or shrinking in capital and earnings in relation to size but without attaining higher efficiency.

Figure 3: On the left, the time-history of the ratio EBIT/NW and σ during five years for firms Campbell Frozen Foods and Lovell. On the right, the respective trajectory drawn in the RRP during the same period. Marks 1 to 7 refer to 1983 to 1987.



The description is complemented with table 1.

Table 1: Mean-adjusted log size σ and the EBIT to NW ratio (1983-1987) for the four mentioned firms. These are the values displayed in the left-hand side of Figures 2 and 3.

Firm	1983	1984	1985	1986	1987
	Mean-adjusted log size (σ)				
CAMPBELL FROZEN FOODS	-0.24	-0.24	-0.25	-0.15	-0.16
LOVELL PLC	-1.09	-1.11	-1.18	-1.15	-1.16
MAUNDER (LLOYD)	-0.26	-0.23	-0.27	-0.31	-0.27
NESTLE (UK)	0.975	1.005	0.998	0.991	0.988
	EBIT to Net Worth ratio				
CAMPBELL FROZEN FOODS	0.218	0.232	0.227	0.100	0.064
LOVELL PLC	-0.01	-0.03	0.020	0.138	-0.12
MAUNDER (LLOYD)	0.026	0.073	0.053	0.091	0.227
NESTLE (UK)	0.152	0.310	0.366	0.538	1.336

NESTLE (UK) and MAUNDER (LLOYD), both in figure 2, show a trend towards higher efficiency. The first firm is large. During the period 1983-1987, it improved from profitability near the industry average to above it. Its Net Worth was kept at level with size. The second firm is small. It recovers from profitability below the average and over-sized, to a new position near average.

CAMPBELL FROZEN FOODS (figure 3) is losing profitability against the industry. The whole of the trajectory lies in the upper two quadrants of the RRP, which means an excess of Net Worth. The second quadrant explicitly means that the firm's Net Worth is not producing the operating profits expected in the Food Manufacturing industry. Figure 3 also shows one firm having negative- and positive-EBIT. LOVELL, despite being a small firm, is over-sized for its Net Worth and for the profits its operations generate.

Benefits From Using the RRP

By comparing the EBIT to NW ratio with the RRP, we note that the former only conveys part of the available information. The RRP is more specific as it also refers to values expected in firms of that size. For example, the profitability of NESTLE (UK) increased during the period. The RRP says as much but also points out that such gain was obtained purely by an increase in efficiency. The proportion of NW and EBIT to size was kept near the industry average. The RRP explained more clearly where the competitive advantage of this firm came from.

Unfavourable positions were more clearly ascertained by the RRP as problems in size, not just in efficiency. During the initial three years of the period, CAMPBELL FROZEN FOODS faced an increase in the proportion of Net Worth to size. Since EBIT also increased in the same proportion, the EBIT to NW ratio was blind to this anomaly. Only too late was the EBIT to NW ratio able to denounce the ensuing plunge in efficiency.

We thus conclude that the complement of the EBIT to NW ratio seems to be useful for the analysis of financial statements and that the RRP seems to make this usefulness more intuitive. Note, however, that the year-to-year adjustment leads most companies to stay close to the origin of the RRP, only noteworthy cases standing out. This is useful for those looking to detect exceptional situations, but an alternative, also interesting possibility would be not to do yearly adjustments, to be able to visualise the trajectories of firms through time.

Automating the Analysis

This section uses mappings to discretize the positions and trajectories of firms in the RRP. It also explains how mapping can be a step towards the automation of ratio analysis.

Statistical models are often used to describe functional relationships. For example, regression lines aim at describing relationships that are linear. However, in some cases it is desirable to describe entire distributions, not just relationships. Statistical tools employed to describe distributions are said to "map" them. Reasons for mapping can be twofold: either the information regarding positions of observations in distributions is relevant and must be preserved, or the density of observations draws a shape that cannot be described functionally.

Both such reasons lead to the use of maps as a way of modelling the RRP. As stressed before, each zone of the RRP is assigned a financial diagnostic. It is the fact that a firm lies in a particular zone that is important for the analysis. Also, the RRP often exhibits irregular shapes. For example, when studying profitability or flow of funds, it is frequent to observe a comet-like shape, that is, a regular density of observations around the origin, except in the third quadrant where a bimodal tail develops. Such shape is difficult to describe functionally.

How to Build Self-Organized Maps

Self-Organized Maps (Kohonen, 1986) are fast and simple Neural Networks able to discretize a multi-dimensional density distribution into a small number of *loci*. In the most common cases, they consist of a one- or two-dimensional lattice of nodes (computation units), each of them capable of computing a given $f(X, W)$ involving the set W of adjustable coefficients (weights) linking that node to the input (independent) variables X. That is, for the j^{th} node, a set of weights $W_j = w_{j1}, w_{j2}, \dots, w_{jM}$ links the input from case k, $X_k = x_{k1}, x_{k2}, \dots, x_{kM}$ to that node. Each node's output, o_j, is dictated by $f(X, W)$, which typically measures some form of similarity between W and X. Often used $f(X, W)$ is the Euclidean distance:

$$o_j = \sqrt{\sum_{i=1}^{M} \left(x_i - w_{ji}\right)^2}$$

The building of the map takes place as follows: all the nodes are supplied with the same input, extracted at random from the firms' data. Then the node with the largest output is found. This node is the one whose weights show greater similarity with that firm. Next, a neighbourhood is defined around this node and the weights of nodes inside it are updated (rewarded) in a way that makes them more like the firm they identified. For example, the new value of weight w_{ji}, linking input i to rewarded node j, can be updated in this way:

$$w_{ji}^{t+1} = w_{ji}^{t} + \eta\left(x_i - w_{ji}^{t}\right)$$

in which t and $t+1$ denote a sequence and η is a small increment. Nodes outside this neighbourhood receive no rewarding. The procedure is repeated for all the firms in the sample and then again and again. At length, the position of firms in the RRP is mirrored by the position of the nodes (as in figure 4) because their weights become like the supplied input. The result is the mapping of the firm's input variables (the RRP in this case) onto a lattice of nodes, each of them characterised by a specific neighbourhood (similar patterns) of input.

How to interpret Self-Organized Maps

In the case of the RRP, the input X are mean adjusted logs of ratios and their complement:

$$X = \{\varepsilon_y - \varepsilon_x, \varepsilon_y + \varepsilon_x\}$$

We refer to nodes as pairs of integers $\{m, n\}$. m is the counter of rows in the lattice and n is the counter of columns. Since each node covers a neighbourhood, the map will be able to identify, for each new firm, the pair $\{m, n\}$ representing a given node. Where a firm's input lies in the neighbourhood of such node, it causes the node to exhibit the largest output (to fire).

Figure 4: Lattice of 9 × 3 nodes superimposed to the respective RRP. Adjacent nodes are connected by straight lines to improve interpretability.

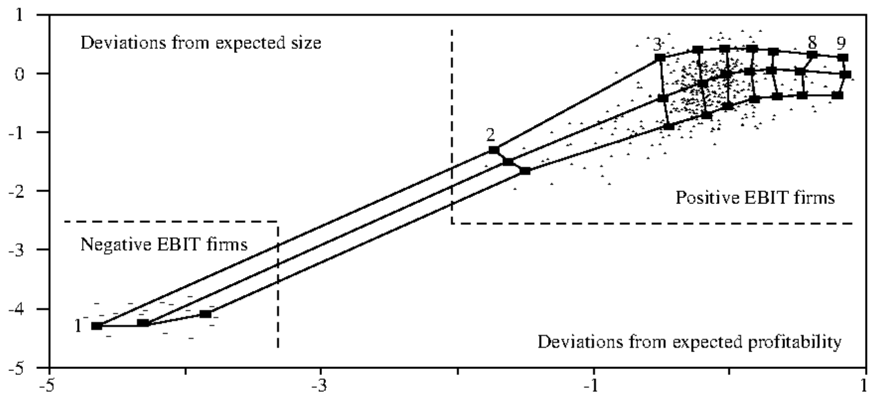


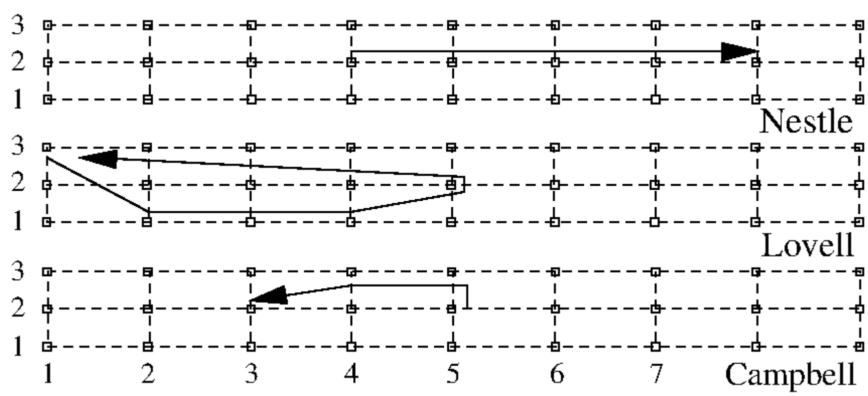
Figure 4 refers to the examples given in the last section. It shows the positions, after the building of the map has finished, of a lattice of nodes superimposed to the density of observations in the RRP. Straight lines link nearby nodes. The discretization performed by the map allows the assigning of a financial diagnostic to each region of the RRP. In fact, since each node acquires a mapping quality, its firing has a precise meaning on financial analytic grounds. This is because each region of the mapped RRP also has a precise meaning. For example, if a firm is shown to a Self-Organized Map and it fires node {2,5}, then this firm conforms with the industry norm, both in efficiency and in size, as the neighbourhood of node {2,5} is the central region of the RRP. It is thus possible to build a 2 by 2 correspondence table relating each node to profitability, and to the magnitude of items denoting profitability when compared with the size of the firm.

Since the RRP uses mean-adjusted log values and the basis of each diagnostic is the extent to which each observation differs from this average, it follows that the table of correspondence between nodes and financial features is expected to be as robust as the median is regarding changes in the variability of statistical distributions of ε_y and ε_x (Laurent, 1963).

After showing this tool a sequence of variables representing a given feature of the same firm during several years, it outputs the corresponding sequence of fired nodes. This output sequence defines a trajectory in the discrete space of the lattice of nodes. Figure 5 presents three of these trajectories. Based on tables of correspondence and on the increments observed in m and n, it would be easy to build an expert system for automatically interpreting these trajectories.

The table of correspondence can be more detailed or less detailed. If, in our example, a larger number of nodes were used in the n dimension, then we would get more specific, albeit less robust diagnostics.

Figure 5: Lattices of nodes showing trajectories of firms in the discrete space. Integer $m, with\ m = 1{,}9$ is the row counter and integer $n, with\ n = 1{,}3$ is the column counter.



Discussion and conclusions

Current developments in information technology can have a fundamental impact on the way the analysis of financial statements is carried out. Our results suggest that a more thorough exploring of possibilities offered by information technology can lead to a more accurate and easier to interpret analysis, together with a better understanding of the statistical characteristics of accounting information.

We have shown that the size-free information discarded by the ratio y/x is the ratio yx/s^2, s being an estimate of the size (scale) effect which is present in all items from the same financial report. We also studied the potential interest of such a complementary ratio in financial statement analysis. Complementary ratios detect deviations from what is expected for a firm of that size. Thus, complementary ratios show useful information about the competitive advantage or disadvantage of a firm. For example, profitable firms may, however, employ more resources (capital plus long-term debt) than other firms in the same industry and are therefore at a disadvantage compared to these other firms. An increase in capital and profits that neither increases nor decreases the efficiency of a firm may become detrimental when that firm has lost the previously existing resource balance relative to size. In sum, complementary ratios detect size-related anomalies, anomalies that standard ratios do not recognise, and provide a measure of size efficiency against which financial features can be measured.

The weak point of the developments presented here is the estimation of the effect of firms' size from the items available in their financial statements. The solution to this problem, as presented in formula (5), is not satisfactory. However, to discuss this matter another paper would be needed.

In this article we also asked to what extent is it useful to gather ratios and their complements in a unique, graphical observation. We remarked that the consideration of both the ratio and its complement can improve the specificity of the analysis and we described a graphical tool, the Rotated Residual Plot (RRP), allowing the study of trajectories. The drawing of trajectories in the RRP reveals a certain behaviour valuable for financial analysis and less explicit when solely using time-histories of ratios. Furthermore, the way the RRP is constructed leads the analyst to naturally focus on noteworthy trajectories, i.e., firms that are positioning themselves advantageously or disadvantageously relative to other firms in the same industry.

The RRP is a different, yet familiar, way of analysing accounting reports. It is different from ratios in that it conveys two pieces of information at a time. But it is based on the same principles: the proportion of one component to the other is supposed to capture features of the firm and the value expected for the industry sets the norm. This tool is not a departure from traditional ratio analysis. Rather, it is its natural extension. All the expertise of ratio analysts can be directly implemented on the RRP. We think that the RRP, when attached to databases of accounting reports, could be an intuitive way of analysing financial statements.

Self-Organized Maps can be used to assign financial diagnostics to regions of the RRP. They can produce robust representations, allowing the relaxing of assumptions about data distributions. And they open the door to machine learning and expert systems.

The implementation of the RRP as a facility attached to databases of accounting reports would require a more effective use of existing technologies. The development of these technologies has presented the accounting profession with an opportunity to take full advantage of computationally more demanding tools.

The impact of the resulting decision support systems is necessarily a matter of conjecture, but the RRP is promising enough to deserve a closer look.

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