Accounting Identities and the Distribution of Ratios

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Abstract

This article describes the influence of accounting identities on the statistical distribution of financial ratios. First, it recalls that where raw numbers are lognormally distributed, then ratios are expected to be positively skewed. Accordingly, the fact requiring an explanation is why some ratios are symmetrical or even negatively skewed, not why the distribution of ratios is positively skewed. Then, the article shows that apparently symmetrical ratios occur because accounting identities act as external boundaries, constraining the long tail of their otherwise skewed distribution to become much smaller. Ratios that are symmetrical or negatively skewed simply reflect the existence of these boundaries. They revert to positive skewness after being inverted, thus making it difficult to accept the hypothesis that the skewness of ratios stems from non-proportionality. Since bounded ratios may induce misleading results when used for calculating confidence intervals or P values, a procedure is suggested to avoid constraints where necessary.

Key-Words: Financial Analysis, Ratio Analysis, Financial Ratio Validity, Financial Ratio Distribution.

JEL codes: C1, C2, C5, C21, G31, M41.

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Introduction

Accounting reports are an important source of information for managers, investors and financial analysts. Ratios are the usual instruments for extracting this information. However, statistical characteristics of ratios pose particular difficulties when used because their distributions vary widely: most ratios are positively skewed but some are symmetrical and others are negatively skewed. Despite the widespread use of ratios in a multitude of contexts, available evidence probably conveys the belief that the distribution of ratios is unpredictable. In fact, researchers and practitioners alike still rely on ad-hoc transformations and outlier deletion in order to adjust the distribution of ratios to approximate normality.

The purpose of this paper is to explain the existence of symmetrical and negatively skewed ratios. Our findings offer guidelines for achieving higher precision when using ratios in a statistical context. The paper is supported by evidence on cross-section distributions of raw numbers and ratios, using data extracted from accounting reports of UK industrial firms. Table 1 shows the accounting identities and the abbreviations used in this paper.

Table 1, page 17

The statistical distribution of ratios has been the object of considerable study. Horrigan (1965), in an early work on this subject, reported positive skewness in some ratios and explained it as a result of effective lower limits of zero. O'Connor (1973) and Bird & McHugh (1977) also found skewness in ratios. Deakin (1976) showed that, in most ratios, positive skewness could not be ignored but also noticed that the ratio TD/TA was near normality. Bougen & Drury (1980) reported skewness, either negative or positive, and extreme outliers. Frecka & Hopwood (1983) extended Deakin's study and reported similar findings. These authors proposed applying transformations and then trimming or winsorising outliers as a means of reaching normality. Ezzamel, Mar-Molinero & Beecher (1987) noticed positive skewness and outliers except for ratios TD/TA and NW/TA and found improvements with square root and logarithmic transformations. So (1987) also found positive skewness except in ratios TD/TA, NW/TA and CA/TA, the latter being negatively skewed. Watson (1990) and Karels & Prakash (1987) studied the multivariate normality of ratios and the advantage of removing multivariate outliers. They noticed that ratios TD/TA and NW/TA were near normality. The same was observed by Ezzamel & Mar-Molinero (1990) who suggested that the trimming of 'obvious' outliers should come first, instead of transforming and then trimming, as proposed by Frecka & Hopwood.

McLeay (1986*a*; 1986*b*) questioned the use by some researchers of such ad-hoc procedures as transformation and trimming of remaining outliers as a means of achieving normality in ratios. He suggested that the data should be left unadjusted and better-fitting models should be used. Tippett (1990) and Rhys & Tippett (1993) developed stochastic processes aimed at identifying the distributional characteristics of ratios.

The paper is organized as follows. The following section lays down the theoretical foundation upon which the paper is based. In summary, we postulate lognormality as the distribution to be found in raw numbers and then we show how ratio components that are perfectly lognormal can produce symmetrical ratios. The next two sections provide the evidence to support the above hypothesis.

The Effect of External Constraints on Skewness

This section first recalls that where raw numbers are lognormally distributed, then the skewness of ratios, as well as the existence of outliers, may be just a general property of multiplicative data. Indeed, the fact requiring an explanation is why some ratios are symmetrical, not why the distribution of ratios is skewed and has outliers. It is proposed that symmetrical or negatively skewed ratios occur because accounting identities act as external boundaries, constraining the long tail of their distributions to become much smaller. Since bounded ratios may induce misleading results when used for calculating confidence intervals or P values, a procedure is offered at the end of this section to avoid constraints.

Theoretical Foundation and Notation

Authors mentioning cross-section lognormality in raw numbers explain it as the outcome of multiplicative processes such as the geometric brownian motion (Tippett, 1990). These processes are considered plausible where raw numbers are accumulations, that is, where they are sums of similar transactions with the same sign (McLeay, 1986*a*).

This study is based upon similar assumptions. However, there is a difference in emphasis. While the authors mentioned above stress the importance of generative mechanisms underlying every item, we focus on the overall effect of size. Instead of assuming that only accumulations such as Sales and Stocks are lognormal, we accept that Sales, Stocks and other items are expected to be lognormal because the growth of the firm as a whole is a stochastic accumulation. Since the effect of size in raw numbers cannot be discarded on *a-priori* grounds, we are inclined to see lognormality as the rule rather than as the exception. The evidence presented later in this paper supports this view.

Where accounting numbers are lognormally distributed, then the logarithm of an observation x_j from financial report j is explained as the expected value of the transformed variable, μ, plus a deviation or residual, e_j. An estimated μ is $\overline{\log x}$, the mean of $\log x$. Ratios y/x can be written as a difference of logarithms:

$$\log y_j - \log x_j = (\mu_y - \mu_x) + (e_y - e_x)_j \quad \text{corresponding to} \quad \frac{y_j}{x_j} = R \times f_j \quad (1)$$

where R is an expected proportion and an estimated R is given by $\exp(\overline{\log y} - \overline{\log x})$, the median of the ratio. Therefore, f_j is, for report j, the percent deviation from the median of the ratio[1]. On a logarithmic scale, this deviation is a difference, $(e_y - e_x)_j$ which we refer to in this paper as $\varepsilon_j^{y/x}$. The distribution of the ratio y/x is the same as the distribution of $f = \exp \varepsilon^{y/x}$. Given that f is an exponentiation of $\varepsilon^{y/x}$, then ratios are expected to be lognormal (Lev & Sunder, 1979; McLeay, 1986*a*).

The peculiar characteristics of lognormal variables and the consequences ensuing from their use must be borne in mind in any context involving the statistical manipulation of ratios. Lognormality cannot be treated as a simple distortion of normality, lognormal variables stem from multiplicative processes while Normal variables are created by additive processes. Of particular interest is the fact that lognormal distributions are very skewed, exhibiting long tails towards positive values. For coefficients of variation[2] beyond 0.25, most of the observations concentrate in a small region with only a few extreme values spreading out over a wide range. It is easy to interpret these extreme values as outliers (Snedecor & Cochran, 1965, p. 281; McLeay, 1986*b*, p. 209; Ezzamel & Mar-Molinero, 1990, p. 13). In fact, outliers often mentioned in relation to ratios are probably just a consequence of multiplicative skewness.

The Distribution of Bounded Ratios

If components of ratios are lognormal, then ratios should be positively skewed. Although most ratios exhibit positive skewness, several authors also mention ratios which are symmetrical or even negatively skewed. As mentioned above, TD/TA and NW/TA have been reported as being Normal and CA/TA has been found to be negatively skewed. How is this possible? The reason seems to be straightforward. Accounting identities make it impossible for some ratios to take on all the values a skewed distribution allows. This constraint is clearly observable when plotting, on a logarithmic scale, the two components of a ratio against each other. Figure 1 compares a constrained ratio with an unconstrained one. The figure shows, on the left, the effect of a boundary imposed by Total Assets on the spread of Net Worth (where the observed values are scattered below the 45^o bisecting line) and, on the right, an unconstrained relationship.

Figure 1, page 18

There is a constraint if, due to an accounting identity or other external cause, the ratio relationship $y_j/x_j = R \times f_j$ is bounded by one of the following inequalities:

$$\text{for any } j, \quad x_j > y_j \quad \text{or} \quad y_j > x_j$$

The inequality on the left can be found in ratios in which the numerator is bounded by the denominator such as Current Assets to Total Assets. The inequality on the right arises in ratios in which the denominator is bounded by the numerator (it is possible to create the latter by taking the inverse or *reciprocal* of the former).

Figure 2, page 19

Figure 2 illustrates the two types of constraint. Where the constraint is $x_j > y_j$, ratios cannot be larger than 1. The effect of this constraint on the distribution of ratios is that it inhibits the spread of its otherwise positively skewed distribution. Instead of the large, lognormal-like tail to the right, such ratios exhibit a smaller one. Where the constraint is $y_j > x_j$, ratio values cannot be lower than 1. The large, lognormal-like tail is unaffected, but the left hand tail is truncated, thus increasing even more the positive skewness of the ratio. In both cases, the bisecting line $x = y$ (or $\log x = \log y$) acts as a boundary. Accordingly, positive skewness would emerge after inverting one of the apparently Normal ratios.

It should be possible to broadly predict the decrease in skewness introduced by a given boundary. Where the constraint is $x_j > y_j$, then $\overline{\log y} - \overline{\log x} < 0$ and $\varepsilon_j^{y/x} < -(\overline{\log y} - \overline{\log x})$ for any j. That is, large positive deviations from the expected value are not allowed. Where the constraint is $y_j > x_j$, then $\overline{\log y} - \overline{\log x} > 0$ and $\varepsilon_j^{y/x} > -(\overline{\log y} - \overline{\log x})$ for any j. That is, large negative deviations from the expected value are not allowed. Since, in both cases, a constraint prevents $\varepsilon^{y/x}$ from being larger than $\overline{\log y} - \overline{\log x}$, then the nearer $\overline{\log x}$ is to $\overline{\log y}$, the stronger the constraint. Thus the difference $\overline{\log y} - \overline{\log x}$ can be used to estimate the extent to which constraints affect the symmetry of the distribution of $\varepsilon^{y/x}$. Taking the spread of $\varepsilon^{y/x}$ into account we obtain the normalized difference

$$\zeta = \frac{\overline{\log y} - \overline{\log x}}{\sqrt{\mathrm{VAR}(\varepsilon^{y/x})}}. \tag{2}$$

In standard deviation units, $|\zeta|$ is the distance separating the constraining boundary from the expected value of the log-ratio. For $|\zeta| > 2$, the constraint is small (less than 2.5% of firms are expected to have their ratios constrained). Thus the lognormal tail or skewness is almost unaffected. For $2 > |\zeta| > 1$, the constraint becomes significant, causing symmetrical or even negatively skewed ratios, as more than 16% of firms are expected to have their ratios constrained.

Besides accounting identities, there are other external factors which may affect the distribution of ratios. However, instead of defining boundaries which are impossible to cross, they bring about a decrease in the density of observations. For example, as firms are likely to avoid negative Working Capital, the inequality $CA > CL$ will influence the density of the distribution of the Current ratio.

Avoiding Constraints

Where the numerator of a ratio is bounded by the denominator, then a simple transformation can take into account the underlying inequality, yielding a new, unbounded ratio. In fact, for any proportion written as $\frac{x_i}{\sum x_i}$ it is possible to calculate the corresponding 'odds ratio', defined as $\frac{x_i}{(\sum x_i) - x_i}$. For example, the odds ratio corresponding to FA/TA is the ratio FA/CA as $CA = TA - FA$. The information contained in both ratios is the same. The difference between odds-like ratios and the corresponding proportion-like ones is just functional. It is therefore possible to avoid ratios affected by constraints by using the corresponding odds ratios instead.

Evidence on Lognormality of Raw Numbers

This section presents an exploratory data analysis supporting the hypothesis upon which the paper is based, providing extensive evidence on the lognormality of raw numbers. Lognormality in items such as Sales, Earnings and Total Assets has received a great deal of attention in texts on the theory of the growth of firms[3]. Since those texts were not oriented towards the analysis of financial statements, they omitted items which are frequently employed as components of ratios, thus failing to supply the kind of evidence required for building the statistical basis of ratio analysis.

McLeay (1986*a*), in one of the few studies contemplating distributions of items as opposed to ratios, argues that items such as Sales, Stocks, Creditors or Current Assets are expected to exhibit cross-section lognormality. Our empirical work confirms this and suggests that the phenomenon of lognormality is much more widespread. Many other positive-valued items have cross-section distributions that are lognormal. Furthermore, where items can take on positive and negative values, then lognormality can be observed in the subset of positive values and also in the absolute values of the negative subset. Size-related non-financial variables such as the number of employees are also lognormal. Our empirical work has also uncovered cases of three-parametric lognormality. This finding may be important for elucidating the origins of non-proportionality in the relationship between the numerator and the denominator of a financial ratio.

Methodology and Data Set

In this study, the lognormality of items was tested by applying two- or three-parametric logarithmic transformations where appropriate. While the Normal distribution is completely specified by the mean and standard deviation, the lognormal distribution may require one extra parameter in order to account for overall displacement of the distribution. Where a displacement of item x (say $x - \delta$), and not x itself, is Normal after a logarithmic transformation, the distribution of x is known as *Three-Parametric* Lognormal. The range of x is thus $\delta < x < \infty$. The usual, *Two-Parametric*, lognormal distribution is a special case for which $\delta = 0$. Since δ is a lower bound for x, it is known as the *threshold* of the distribution (Aitchison & Brown, 1957). The normality of the transformed observations was assessed using an improved version of the Shapiro-Wilk test (Royston, 1982)[4]. This test can cope with large or small sample sizes and is recommended as a superior omnibus test. Notice that the subtraction of δ from x is not similar to the practice of adding a constant value to observations for avoiding negative values (Ezzamel & Mar-Molinero, 1990) as the subtraction of δ never changes the sign of observations.

The data set used in this study was taken from the Micro-EXSTAT database for five consecutive years (1983-1987). Following Sudarsanam & Taffler (1985), we extracted 14 industries to be used as intra-industry samples (table 2) and we also pooled all the extracted firms into a single cross-industry sample. Only UK firms were selected. Both intra-industry and cross-industry groups were examined. The number of firms per industry ranges from a minimum of 13 (Leather, 1983) to a maximum of 145 (Electronics, 1986). The number of firms in the cross-industry samples ranges over the years from 550 to 702. Where a sample contained sufficient negative values, two separate tests of lognormality were performed by taking the subset of positive values and then the absolute values of the negative subset. This is because cross-sections of positive and negative values should be analysed separately as they may be seen as different populations (Lev & Sunder, 1979).

Table 2 also shows the accounting variables tested. These are frequently employed as components of ratios. A further variable included in the analysis is the number of employees (N) which allows the comparison with a non-accounting variable exhibiting similar statistical characteristics.

Intra-Industry Results

In the examination of individual industries, 1,260 tests were carried out, corresponding to 18 different items for each of the 14 selected industries, during a period of 5 years. Lognormality could not be rejected in most of these samples, as follows:

- two-parametric lognormality could not be rejected in 1104 tests (87.6%);
- three-parametric lognormality could not be rejected in 136 tests (10.8%);
- the hypothesis of lognormality was rejected in 20 tests (1.6%).

Table 2, page 20

Table 2 shows these results in more detail. It displays the number of years in which two-parametric lognormality was rejected for each industry and item. Numbers with asterisks indicate rejection of lognormality. Numbers without asterisks indicate acceptance of three-parametric lognormality. For instance, 2+1* in column 'CL' and row 'Electronics' indicate that Current Liabilities in the Electronics industry was three-parametric lognormal in two years of the period 1983-1987 and was not lognormal in one of the remaining years.

The results summarized in table 2 suggest that the industrial grouping mostly determines whether samples are expected to be two- or three-parametric lognormal. Industries are more important than items in explaining significant thresholds: 21% of the examined industries account for 65% of cases of three-parametric lognormality. This table also shows that the rejection of two-parametric lognormality is sporadic: only in one sample, Wages in the Electronics industry, do rejections persist during five years.

The 20 tests rejecting lognormality (table 2) were closely observed. Extreme outliers, clearly erratic, were found in 7 of them. The other 13 cases belong to the Food Manufacturing and Electronics industries, exhibiting well detached clusters of firms.

Only two industries (Electronics and Food Manufacturing) contained enough negative values to allow the testing of the absolute values of the negative subset. In contrast with the positive values, two-parametric lognormality was prevalent.

Cross-Industry Results

The results of testing the pooled samples for lognormality are also displayed in table 2 (below). Lognormality was not rejected for any item in any year. Eleven items were found to be two-parametric lognormal during the whole period 1983-1987. The remaining 7 items were either two- or three-parametric depending on the year. Similar results were obtained for the absolute values of the negative subset: $EBIT$ and FL were, in one or two years, three-parametric.

An additional finding that is worth reporting, and that applies to the industry samples as well as the pooled cross-industry samples, concerns the positive kurtosis observed in all cases after transformation. The skewness and kurtosis of the raw data were extreme, as expected. After a logarithmic transformation, the skewness vanished but all of the samples continued to exhibit traces of leptokurtosis.

Testing Other Transformations

This study also tested the possibility of obtaining normality when using transformations other than the logarithmic. The logarithmic transformation can be viewed as a way of removing positive skewness. It makes sense to ask whether the achieved reduction in skewness is appropriate. If less reduction is required, a square root or another root should be used instead. If more reduction is required, the Pareto distribution or another of its class should be used.

First, a scale of roots progressively approaching the effect of a logarithmic transformation was tested. We observed that there is progress towards normality for roots of increasing exponent and that symmetry is maximal when using logarithms. It may be noted that Ezzamel & Mar-Molinero (1990) reported an unpredictable distribution of ratios after applying similar transformations, which contrasts with the regularity observed in the underlying accounting variables.

Figure 3, page 21

The Pareto transformation, a more powerful transformation than the logarithmic in neutralizing positive skewness, was also tested. Pareto distributions occur if the relationship between observations and their rank in the sample is logarithmic. Where values are ranked from large to small, then log-values and log-ranks should be linearly related for the Pareto to hold[5]. However, a clear downward concavity of the distribution was observed in all tests. In general, firms occupying the middle of the rank were found to be about twice as large as that predicted by the Pareto distribution. Figure 3 shows an example of the relationship between logarithms of Creditors and logarithms of their rank. Observations follow much more closely the lognormal deviate (the dashed line) than a Pareto straight line. Ijiri & Simon (1977) reported the same concavity for US data.

Comparing Bounded and Unbounded Ratios

This section compares bounded and unbounded ratios, stressing their different characteristics. Two sets of ratios are identified. In the first set, the denominator is a boundary to the numerator. The skewness of these ratios is smaller than expected in multiplicative data, suggesting that symmetrical or negatively skewed ratios reflect the existence of boundaries. Moreover, bounded ratios become skewed after being inverted, thus making it difficult to accept the hypothesis that the skewness of ratios stems from nonproportionality. In the second set of ratios, the denominator is not likely to bound the numerator. In this set, the estimates of skewness are in agreement with values expected for multiplicative data, showing that its origin is lognormal.

Besides classifying ratios as bounded or unbounded, the criterion adopted for selecting ratios was twofold: ratios in both sets should share as many items as possible and they should resemble those already tested by other authors. Five years (1983-1987) were examined. Only positive values were included since, as mentioned above, cross-sections of positive and negative values may be seen as different populations.

Bounded Ratios Are Near Symmetry

Positive skewness should decrease in proportion to the strength of constraints affecting ratios. The nearer the numerator of these ratios is to the denominator, the farther should their distributions be from positive skewness. In order to test this hypothesis, 14 ratios were selected in which the numerator is bounded by the denominator. For each of them, $|\zeta|$ in formula (2) was calculated.

Table 3, page 22

Table 3 displays the selected ratios, their skewness and the value of $|\zeta|$. The values observed for skewness agree with those reported by other authors. As shown above, normalized distances below 2 denote significant constraints. Ratios CA/TA and QA/CA should be the most affected, as their $|\zeta|$ is small. In fact, these ratios exhibit negative skewness. Probably, this is because they are so strongly constrained that their distributions become skewed in the negative direction. Lognormal distributions are two-tailed. If the large right hand tail almost vanishes, the small left hand tail introduces negative skewness.

The values of $|\zeta|$ suggest that ratios like NW/TA or I/CA should be significantly affected, though less than those mentioned above. In fact, these ratios are almost symmetrical. This is probably because the large right hand tail of their distributions is shortened to an extent where it is in balance with the left hand tail. Next, FA/TA or I/TA are affected to a smaller degree. Their skewness is positive but less than expected in multiplicative data. Finally, given $|\zeta|$, the constraint should be very small in ratios like C/TA or $DEBT/TA$ and almost non-existent for $EBIT/S$. In fact, this reasoning is supported by the data as these ratios are very skewed.

Table 4, page 23

Unbounded Ratios Are Broadly Lognormal

The 15 ratios for which there is no obvious constraint, are listed in table 4, with their reciprocal. Three facts emerge. First, these ratios are not far from lognormality. This can be ascertained by observing the strict relationship between their skewness and kurtosis, which is a typical feature of multiplicative data (Aitchison & Brown, 1957, pp. 8-9).

Figure 4, page 24

Figure 4 is a graphical representation of table 4. It displays the regular curve that is formed by unconstrained ratios when skewness is plotted against kurtosis. Each ratio is represented by a plus sign. This regularity is close to the relationship expected in lognormal variables, as indicated by the dashed line[6]. Ratios exhibiting larger skewness and kurtosis display a small, systematic drift from the theoretical curve.

Second, the inference that profitability ratios such as $EBIT/TA$ or $EBIT/NW$ are multiplicative would appear to contradict the findings of some other authors. The literature on the distribution of ratios seems to implicitly consider profits as additive, albeit non-normal (McLeay, 1986*a*; 1986*b*; Tippett, 1990; amongst others). Probably this is because, when studying profitability ratios, negative values are included in samples. However, according to our assumptions, where raw numbers take on positive and negative values across firms, then the distribution of negative values should be a negative mirror-image of the lognormal distribution. In that case, the overall distribution of items such as Earnings or Working Capital would be a juxtaposition of two lognormals. Ratios formed with these combined distributions might be markedly two-tailed. Long-tailed distributions such as Student's t or Cauchy (McLeay, 1986*b*) could fit them closely.

A third fact about unbounded ratios is that none of them is exactly lognormal, despite the strict lognormality of raw numbers. Lev & Sunder (1979, p. 204) and McLeay (1986*a*), when noticing that ratios of lognormal variables should also be lognormal, were referring to the theoretical case. Ratios are near lognormality but their logarithms are leptokurtic. Log-leptokurtosis is also observed in intra-industry ratios and in ratios formed with non-accounting items such as the number of employees. The presence of leptokurtosis in log-ratios explains why, for some ratios, no transformation seems to suceed in approximating normality (Beaver, 1966; Ezzamel *et al.*, 1987; Ezzamel & Mar-Molinero, 1990).

Skewed Ratios and Non-Proportionality

The main reason for using ratios is to remove the influence of firm size from accounting variables. In the course of their critiques of the practice of ratio analysis, Lev & Sunder (1979) and Whittington (1980) argued that size is only properly removed where the numerator and the denominator of the ratio are proportional. Accordingly, these authors advocated a regression rather than a ratio approach to remove the effect of size. Barnes (1982), added that non-proportionality probably also explained why the distribution of ratios is skewed. These views were shared by Lee (1985), Ezzamel *et al.* (1987), So (1987) and others. A continuing stream of research on the validity of ratios routinely implies that non-proportionality may have a role in explaining distributions of ratios.

However, since ratios are multiplicative and skewness is an expected quality of multiplicative data, then non-proportionality is not required for explaining skewness in ratios. In fact, if skewness were caused by non-proportionality, then ratios which are symmetrical should also be proportional. They should obviously remain proportional and symmetrical when inverted. The constraining mechanism predicts the contrary: reciprocals of symmetrical ratios should be very skewed.

Table 5 compares the skewness of CA/TA, NW/TA and FA/TA[7] with the skewness of their reciprocals. It can be seen that the reciprocals are distinctly multiplicative while the original ratios are not far from normality.

Table 5, page 25

Concluding Remarks

This paper raises two important issues. First, the widespread lognormality of raw accounting numbers suggests that the mechanism governing their cross-section distribution is general rather than particular to this or that item. In order to explain lognormality in accounting numbers, it might be sufficient to consider the growth of the firm as multiplicative with accounting variables reflecting, on average, a given proportion of firm size. Second, functional relationships between two lognormal variables may describe an expected proportionality of random effects, of which strict proportionality is just the simplest formulation. Therefore, besides ratios, other functional forms exist, capable of modelling the statistical characteristics of accounting numbers while removing the effect of size. For example, three-parametric lognormality suggests an obvious extension of ratios probably able to comprise non-proportionality.

It may be concluded that this paper removes one major difficulty in understanding the statistical distribution of ratios: the existence of symmetry and negative skewness is explained as the effect of external boundaries such as accounting identities. The widespread lognormality of raw accounting numbers provides a privileged viewpoint from which ratios can be studied. First, it shows that ratios are expected to be multiplicative. Thus deviations from positive skewness, not deviations from symmetry, should be the main object of interest. Second, it unveils interesting features of ratios such as log-leptokurtosis. The findings of this study show that, after all, there is something regular and easy to understand in ratios.

Notes

1. Lev & Sunder (1979, p. 191) also mention multiplicative residuals.
2. The coefficient of variation is the standard deviation expressed as a fraction of the expected value. It is preferable to the standard deviation or the variance for quantifying the spread of lognormal data, because the latter are not constant.
3. See Ijiri & Simon (1977), for example.
4. For each test, δ was estimated by applying a modified version of the procedure suggested by Royston (1982, p. 123). The Shapiro-Wilk test produces a statistic, W, ranging from zero to one. Values of W approaching 1 mean increasing normality. Royston uses trial and error to find out which δ maximizes W. Using simulation, we noticed that the threshold should be estimated as the smallest δ able to attain a non-significant W, not as the δ yielding the largest W.
5. Where x is Pareto-distributed, then $\log x = \log M - \beta \times \log r$. r is the rank of x. The largest x is assigned the rank 1. M and β are parameters of the distribution.
6. The SPSS-X utility used in this study computes skewness and kurtosis in a way that is different from the conventional definition. For details, see SPSS Inc., (1983). *SPSS^x Statistical Algorithms*, Chicago, Il.
7. The distribution of FA/TA is the mirror-image of CA/TA. This is because the two ratios add to 1. The same can be observed in the pair I/CA and QA/CA.

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TA	Total Assets	NW	Net Worth
FA	Fixed Assets	DEBT	Long Term Debt
D	Debtors	C	Creditors
CA	Current Assets	CL	Current Liabilities
I	Inventory	TC	Total Capital Employed
WC	Working Capital	TD	Total Debt
EX	Operating Expenses less Wages	S	Sales
EBIT	Earnings Before Interest and Tax	W	Wages
OPP	Operating Profit	QA	Quick Assets
FL	Gross Funds From Operations	N	Number of Employees

Table 1: List of abbreviations used in this study.

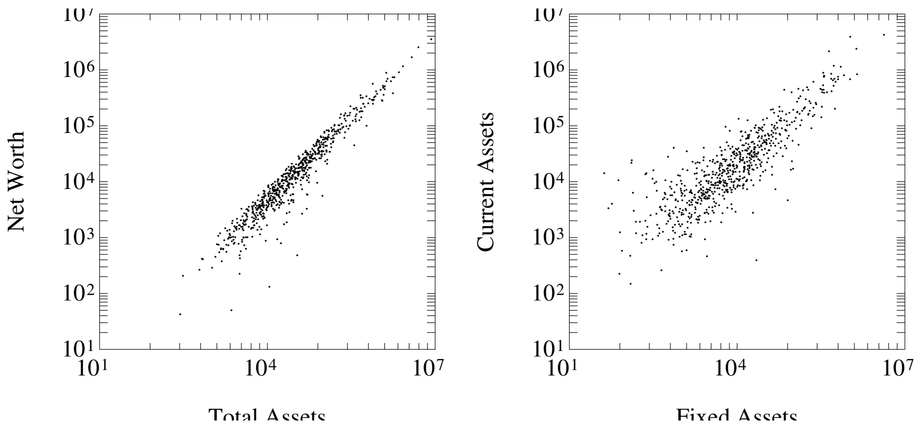

Figure 1: Two scatterplots comparing constrained (left) with unconstrained (right) bivariate distributions of raw numbers. Each dot is one firm. The axes use logarithmic scaling. The constraining frontier is the bisecting line $\log x = \log y$.

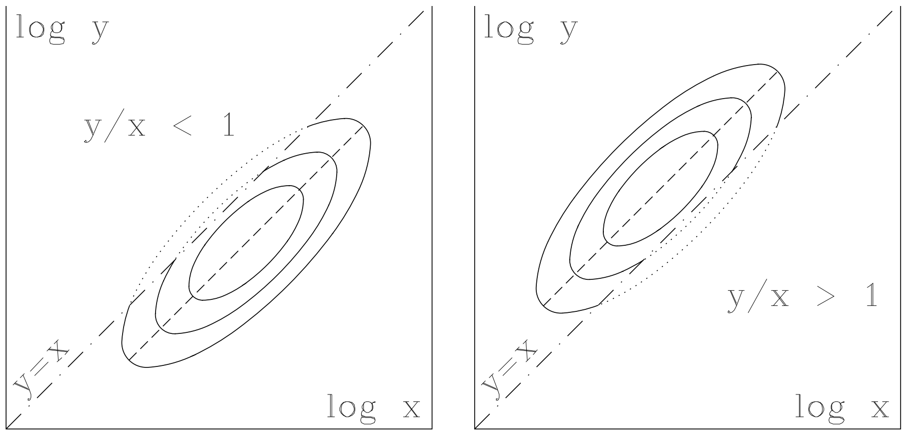

Figure 2: The two kinds of constraints affecting bivariate distributions of raw numbers. The axes use logarithmic scaling.

										Raw Item:									Total by
Industry:	FA	EBIT	I	OPP	D	WC	NW	C	W	EX	CA	DEBT	TC	N	CL	TA	FL	S	Industry
Misc. Textiles																			0
Leather																			0
Metallurgy																			0
Motor Componen.												1					1		2
Wool						1						1							2
Machine Tools			1		1						1			1					4
Building Mater.	2	1					1										1		5
Clothing				1*						1*							1+1*	1*	1+4*
Paper & Pack.					2	1				2+1*	1*							1	6+2*
Chemicals	1		1					1				2			3		1		9
Electrical			3				2		1		2		4	2		3	1	1	19
Indust. Plants	2	1	1*	2	2	2	2	2	1	1	1	1	2	3	2	2	1	4	31+1*
Electronics		2		2	2	2	2	1	2+3*		2		1+1*	1+3*	2+1*	3	1	3	26+8*
Food Manufact.		1+1*		1		1	1	4	2	3+1*	1+1*	3+1*	2		3	3	3	3+1*	31+5*
Total by Item	5	5+1*	5+1*	5+1*	7	7	8	8	6+3*	6+3*	7+2*	8+1*	9+1*	7+3*	10+1*	11	10+1*	12+2*	136+20*
Inter-Industry (+)								2	4		2			2	2	4	1		17
Inter-Industry (−)		2															1		3

Table 2: Number of years in which tests led to the rejection of two-parametric lognormality by industry and by item.

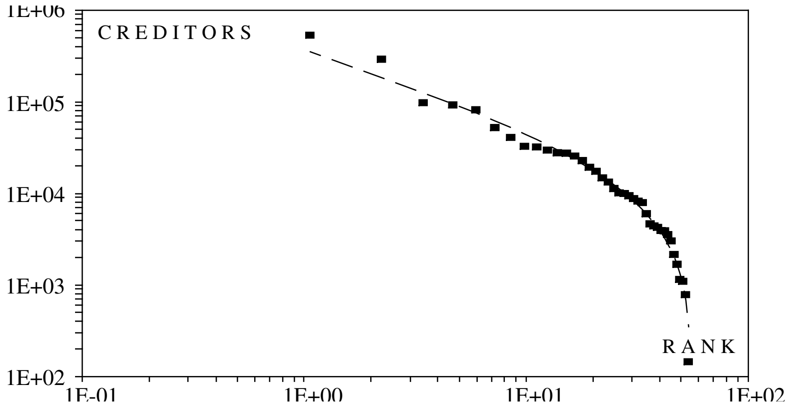

Figure 3: Relationship between log-rank (X-axis) and log-value (Y-axis). The dashed line is the lognormal deviate. Each point represents the Creditors item reported by a firm in the Building Materials industry, 1987.

Ratio	1983		1984		1985		1986		1987	
	skew	zeta	skew	zeta	skew	zeta	skew	zeta	skew	zeta
CA/TA	-0.41	1.29	-0.43	1.05	-0.35	1.58	-0.50	1.34	-0.59	1.14
CL/TA	0.64	2.2	0.59	1.84	0.62	2.21	0.56	2.24	0.59	2.26
C/CL	-0.09	1.54	-0.15	2.02	-0.21	1.56	-0.27	1.63	-0.27	1.72
C/TA	1.25	2.88	1.23	2.61	1.36	2.94	1.19	2.87	1.14	2.96
DEBT/TA	1.87	2.37	1.92	2.29	2.1	2.36	1.77	2.17	2.06	2.21
FA/TA	0.41	1.69	0.43	1.96	0.35	1.76	0.5	1.76	0.59	1.81
I/CA	0.26	1.53	0.1	1.74	0.22	1.57	0.43	1.44	0.29	1.33
I/TA	0.57	2.08	0.47	1.9	0.87	2.16	0.91	2.04	0.81	1.89
NW/TA	-0.15	1.61	0.01	1.85	0.01	1.73	-0.01	1.8	-0.06	1.82
QA/CA	-0.26	1.34	-0.1	1.17	-0.21	1.81	-0.43	1.4	-0.29	1.3
QA/TA	0.5	2.07	0.58	1.92	0.51	2.41	0.39	2.17	0.45	2.26
TD/TA	0.35	2.31	0.17	2.19	0.22	2.23	0.19	2.17	0.18	2.21
EBIT/S	2.03	3.06	1.63	3.16	2.06	3.12	1.82	3.18	1.48	3.25
W/S	0.42	2.14	0.42	2.01	0.41	2.08	0.39	2.13	0.37	2.13

Table 3: $|\zeta|$ (zeta) and skewness for 14 ratios likely to suffer constraints.

Ratio	1983		1984		1985		1986		1987	
	skew	kurt	skew	kurt	skew	kurt	skew	kurt	skew	kurt
D/C	5.7	44.5	5.6	48.3	14.0	255	15.0	261	4.8	35.5
C/D	18.7	381	20.5	461	22.5	535	14.3	249	12.4	198
CA/CL	2.0	5.8	4.2	29.2	5.0	38.4	11.6	205	4.8	43.6
CL/CA	19.6	415	4.2	34.5	3.0	22.9	9.2	135	21.6	506
C/I	9.9	139	9.8	132	24.2	592	15.8	268	21.3	491
I/C	6.5	71.5	3.3	18.0	8.9	119	9.2	119	7.9	91.3
S/TA	7.0	72.3	13.6	238	13.3	241	6.3	51.7	8.5	95.3
TA/S	8.8	111	19.5	436	7.6	89.0	15.9	336	3.4	22.3
S/FA	9.8	105	21.4	486	10.4	115	9.2	96.1	9.6	103
FA/S	2.3	9.4	15.9	317	8.9	125	5.1	50.8	5.9	60.9
S/NW	16.4	310	13.2	188	23.0	549	16.1	324	16.5	207
NW/S	1.6	5.2	4.4	38.6	2.5	17.6	12.0	223	4.7	39.8
S/I	11.7	168	12.0	189	17.0	306	23.5	571	21.3	481
I/S	8.9	132	17.1	358	17.9	388	9.8	179	1.4	4.5
EBIT/TA	2.0	8.3	2.4	11.2	1.4	3.6	1.9	7.3	1.2	2.6
TA/EBIT	16.6	291	12.2	172	11.1	173	7.5	76.0	18.3	374
EBIT/NW	19.9	425	17.5	317	16.6	322	8.4	110	16.4	330
NW/EBIT	15.7	256	17.6	369	12.7	227	7.0	71.9	23.4	567
EBIT/FA	9.4	117	11.2	170	10.4	155	9.0	107	8.5	98.8
FA/EBIT	12.1	154	9.8	126	11.6	156	7.7	83.8	16.8	321
D/I	10.4	127	16.6	329	21.6	578	24.4	618	19.9	423
I/D	10.7	128	11.8	166	16.0	298	10.0	127	7.5	74.6
W/I	15.8	268	9.3	116	7.9	79.0	13.6	204	14.4	219
I/W	8.6	89.7	4.6	28.1	4.5	31.2	3.1	12.7	3.3	14.0
W/TA	1.3	4.9	1.5	5.8	1.4	4.3	1.8	7.2	1.5	6.4
TA/W	9.5	107	11.5	153	5.7	50.6	4.2	27.0	3.7	19.7
DEBT/NW	14.5	228	10.9	140	8.6	90.7	7.5	78.0	5.4	37.4
NW/DEBT	13.3	200	17.6	333	12.0	185	11.0	146	16.1	296

Table 4: Skewness and kurtosis of 15 unconstrained ratios (positive observations only). All industries together.

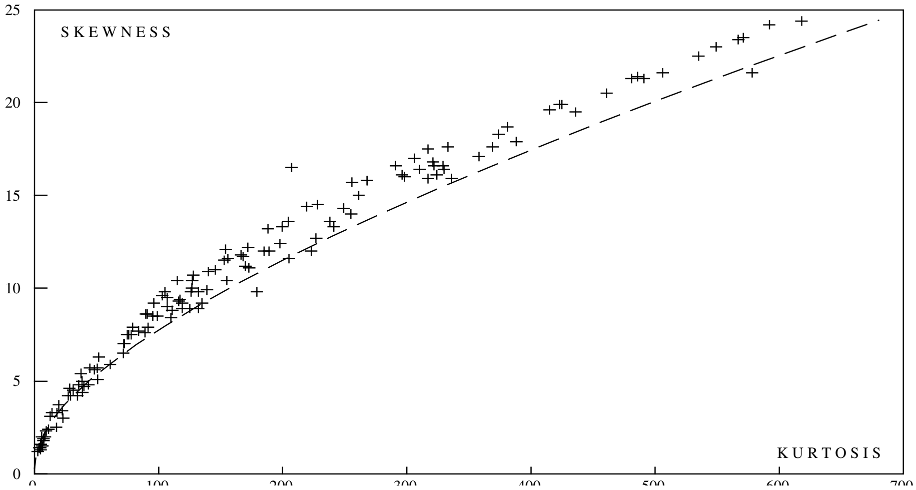

Figure 4: The functional relationship between skewness and kurtosis in unconstrained ratios. The dashed line is the theoretical relationship. Each ratio is represented by a plus sign.

Ratio	Not inverted					Inverted				
	1983	1984	1985	1986	1987	1983	1984	1985	1986	1987
CA/TA	-0.41	-0.43	-0.35	-0.50	-0.59	18.7	5.04	6.29	20.7	17.9
NW/TA	-0.15	0.01	0.01	-0.01	-0.06	17.7	15.4	22.3	12.9	12.4
FA/TA	0.41	0.43	0.35	0.50	0.59	11.2	21.9	23.8	12.7	11.2

Table 5: Skewness of three constrained ratios and their reciprocal. All industries.